FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 3 Round 1
Arithmetic: Scientific & Base Notation

1.)

2.)

3.)

1.) If $\frac{(10^{x+2})(100^{x-1})}{\left(\frac{1}{1000}\right)^{x}} = 10^{\{42,48,54\}}$, find the value of x.

2.) Let a be a positive integer, and suppose that $101_{a+1} - 21_a = \{170_{10}, 226_{10}, 145_{10}\}$. What is 11_a expressed in base 10?

3.) Hexadecimal (base 16) notation has 16 digits: 0-9, as well as A = 10, B = 11, C = 12, D = 13, E = 14, and F = 15. If $\{10{,}40{,}20\}, x, y,$ {6.C,F.A,7.D} are four consecutive terms of a geometric sequence in hexadecimal, find the value of x. Enter it as a number in base 3 notation.

FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 3 Round 2 Algebra 1: Word Problems

1.)

2.)

3.)

1.) Ivanna Hybrid commutes to work from home every day. On Monday her average speed in the morning is 50 miles per hour, but on the way home due to traffic she only manages an average speed of 40 miles per hour. If her total commute time on Monday was $\{\text{two hours and forty-two minutes, two hours and fifteen minutes, three hours and nine minutes}\}$, find the distance from her home to her work in miles.

2.) The formula for converting a temperature in degrees Celsius (C) to its exact equivalent in degrees Fahrenheit (F), is $F = \frac{9}{5}C + 32$. A popular approximate way of performing the same conversion is phrased as "double it and add 30". If on a given day the approximation gives a temperature in degrees Fahrenheit that is $\{2,3,4\}$ degrees Fahrenheit higher than the actual temperature, find the actual temperature in degrees Fahrenheit.

3.) A large bowl contains only peanuts and cashews. Initially, the bowl contains a total of $\{30,45,35\}$ cups of nuts. Then $\{6,4,4\}$ cups of peanuts and $\{9,11,9\}$ cups of cashews are added to the bowl. As a result, the quantity $\frac{\text{number of cups of peanuts}}{\text{number of cups of cashews}}$ is now one half of what it was previously. How many cups of peanuts were in the bowl originally?

FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 3 Round 3
Geometry: Polygons

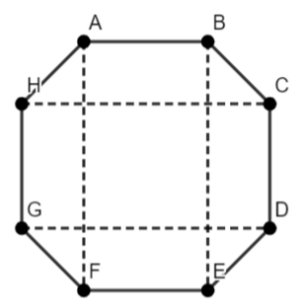
1.)

2.)

3.)

1.) How many diagonals are in a polygon whose interior angle measures in degrees total $\{1800,1980,2160\}$?

2.) The diagram (not drawn to scale) shows octagon $ABCDEFGH$. $AB = CD = EF = GH = 1$ and $BC = DE = FG = HA = x\sqrt{2}$. Each interior angle of the octagon measures 135°. The area of the octagon is $\{3,5,9\}$. Find $100x$ rounded to the nearest integer.



3.) A regular m-gon and a regular n-gon, with $m < n$, have the property that the difference between the measure of one interior angle of each is $\{6,8,9\}$ degrees. Find the largest possible value of n.

FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 3 Round 4
Algebra 2: Functions & Inverses

1.)

2.)

3.)

Note: the inverse f^{-1} of a function f is not necessarily a function.

1.) If $f(x) = 2x + 5$ and $f^{-1}\left(f^{-1}(a)\right) = \{3,4,5\}$, find the value of a.

2.) Let k be a rational number that can be expressed in simplest form as $\frac{a}{b}$ where a and b are integers. If $g(x) = x^3 + 3x - 10$ and $g^{-1}(\{10k^2 - 6k + 5, 14k^2 + 2k - 3, 36k^2 + 2k + 8\}) = 2k$, find $\frac{1}{2}ab - b$.

3.) Let $f(x) = a\sqrt{x+b} + c$, where a, b, and c are real constants. If $2f(4x + 1)$ has a domain of $x \geq -\frac{9}{16}$ and a range of $y \leq 100$ and $f(1) = 41$, find $\{f(5), f(11), f(19)\}$.

FAIRFIELD COUNTY MATH LEAGUE 2020-2021

Match 3 Round 5 Precalculus: Exponents & Logarithms

1.)

2.)

3.)

1.) If a and b are positive integers and $\frac{5^a * 2^b}{5^{10} * 2^3} = 8 * 10^{\{300,100,200\}}$, find the value of $a + b$.

2.) If $\log_{\{3,5,7\}}\left(1 - \frac{1}{2}\right) + \log_{\{3,5,7\}}\left(1 - \frac{1}{3}\right) + \cdots + \log_{\{3,5,7\}}\left(1 - \frac{1}{n}\right) = -4$, find the value of n.

3.) If $\log_3\{45,135,405\} = x$ and $\log_4\{400,1600,100\} = y$, then $\log_3 4 = \frac{ax-b}{cy-d}$ for positive integers $a, b, c,$ and d, where a and c are relatively prime. Find the value of $a + 2b + 3c + 4d$.

FAIRFIELD COUNTY MATH LEAGUE 2020-21

Match 3 Round 6 Miscellaneous: Matrices

1.)

2.)

3.)

1.) If $\begin{bmatrix} a & a \\ c & c \end{bmatrix} + \{2,3,4\}\begin{bmatrix} 6 & b \\ b & 1 \end{bmatrix} = \begin{bmatrix} 15 & 33 \\ 50 & d \end{bmatrix}$, find the value of d.

2.) If $A = \begin{bmatrix} 4 & 2 \\ 7 & x \end{bmatrix}$ and $\det(2A^{-1}) = \left\{\frac{2}{5}, \frac{2}{9}, \frac{2}{13}\right\}$, find the value of x.

3.) If $A = \begin{bmatrix} x & 8 \\ y & z \end{bmatrix}, B = \begin{bmatrix} 9 & -1 \\ 3 & 4 \end{bmatrix}, x > 0$, $AB = BA$, and $\det(A) =$ $\{321, 417{,}576\}$, find the value of z.

FAIRFIELD COUNTY MATH LEAGUE 2020-21 Match 3 Team Round

1.) Mike and Andrew rake leaves. It would take Andrew twenty minutes longer to rake a particular lawn than it would take Mike. Mike starts raking alone. After twenty minutes Andrew joins him, but after another twenty minutes Andrew gets bored and quits. It takes Mike another 24 minutes to finish raking the lawn alone. Assuming that each person's raking did not interfere with the other's, how many minutes would it have taken Mike to rake the lawn alone?

2.) For a particular value of k, the function $f(x) = \frac{\sqrt{x}-k}{x^2-9}$ has a range of $(0, a) \cup (a, b]$, where a and b are positive real numbers. Find the value of $\frac{1}{ab}$.

3.) Matrix A has the property that the entries in each row form a unique arithmetic sequence. If $B = \begin{bmatrix} 3 & 1 \\ 6 & -4 \\ 2 & 3 \end{bmatrix}$ and $AB = \begin{bmatrix} 52 & 6 \\ 112 & -4 \end{bmatrix}$, find the sum of the elements of A.

4.) Assuming the pattern continues, find the following product of 100 logarithms: $\log_5(25^2) \log_{25^2}(125^3) \log_{125^3}(625^4) \ldots$.

5.) The number 2020! has 5802 digits. If 2020! were written in scientific notation as $a * 10^b$ where a is a decimal between 1 and 10 with n digits (and no trailing zeros) and b is a positive integer, find the value of $n + b$.

6.) An n-gon has d diagonals, and, in a regular d-gon, each interior angle has an integer degree measure. How many different possible values of n are there?