FAIRFIELD COUNTY MATH LEAGUE 2019-2020

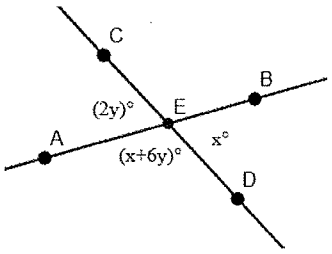
Match 6 Round 1
Arithmetic: Lines & Angles

1.) 36°

2.) 16 minutes, 22 seconds

3.) 135° or 164°

1.) Consider the diagram to the right (not necessarily drawn to scale) of lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersecting at point E. Find $m\angle BED$ in degrees.



2.) After an analog clock strikes 12:00, what is the fewest number of minutes and seconds that must pass before the hands form a 90-degree angle? Round your answer to the nearest whole number of seconds.

3.) Consider kite $ABCD$ with $AB = AD$ and $BC = CD$. If $m\angle C = (3x + 150)^\circ$, $m\angle A = (x^2)^\circ$, and $m\angle B = 4m\angle A$, find all possible values in degrees of $m\angle C$.

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Match 6 Round 2
Algebra 1: Literal Equations

1.) $n = \frac{FV}{ART}$

2.) $c = \frac{ad}{x} - bd$ or $c = \frac{ad-bdx}{x}$

3.) $x = \frac{3}{y} + 2$ or $x = \frac{2y+3}{y}$

1.) If $PV = nRT$ and $F = PA$, find n in terms of V, F, A, R, and T.

2.) If $x = \frac{a}{b+\frac{c}{d}}$, find c in terms of a, b, d, and x.

3.) If $x \neq 2$ and $y \neq 0$, find x in terms of y given $x^3y + 4xy - 12 = 2x^2y + 3x^2 + 8y$.

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Match 6 Round 3
Geometry: Solids & Volume

1.) $\pi\sqrt[3]{36}$

2.) 8

3.) $\frac{1}{4}$

1.) What is the surface area of a sphere with a volume of π?

2.) The center of a given cube is one unit away from any vertex. Find the surface area of the cube.

3.) A solid cylinder of radius r and height h is sliced in half parallel to either base, and each resulting smaller solid cylinder has $\frac{3}{5}$ the surface area of the original cylinder. Find $\frac{r}{h}$.

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Match 6 Round 4
Algebra 2: Radical Expressions & Equations

1.) $\frac{\sqrt{6}}{3}$

2.) $x = \frac{4}{3}$

3.) $x = -\frac{15}{32}$ or $x = 40$

1.) Express in simplest radical form: $\frac{8^{5/6} \cdot 27^{1/6}}{12}$

2.) Solve for all real values of x: $\sqrt{x} + \sqrt{3} = \sqrt{x+7}$

3.) Solve for all real values of x: $4(2x+1)^{3/4} - \sqrt[4]{2x+1} = 12\sqrt{2x+1} - 3$

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Match 6 Round 5 Precalculus: Polynomials & Advanced Factoring

1.) $(x+1)(x+6)(2x-5)$

2.) 1 or 17

3.) $(-18,12)$

1.) Factor into three binomials with integer coefficients: $2x^3 + 9x^2 - 23x - 30$.

2.) The remainder when $f(x) = x^2 + 2kx + 3k^2$ is divided by $x + 2$ is 8. Find all possible values of $f(1)$.

3.) The polynomial $f(x) = x^4 + ax^2 + bx + 80$ with real coefficients a and b has a zero of $x = 3 - i$. Find the ordered pair (a, b).

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Match 6 Round 6 Miscellaneous: Counting & Probability

1.) 90720

2.) $\frac{41}{44}$

3.) $\frac{24}{49}$

1.) How many unique arrangements of the letters of "fairfield" are there?

2.) A group of three students is to be chosen at random from a club consisting of 3 seniors, 4 juniors, and 5 sophomores. Find the probability that at least two students in the group of three will be from different grades.

3.) A math league coach wants to incentivize her students to perform well in Match 6. She allows her two highest scorers to select one of two identical un labeled boxes at random and reach in and select a prize at random from that box. Box A contains 6 Panera gift cards and 4 Google Play gift cards. Box B contains 4 Panera gift cards and 5 Google Play gift cards. The first student selects a box at random and pulls out a Panera gift card. The second student selects the same box and pulls out a Google Play gift card. What is the probability the two students selected box A?

Team Round .

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1.) $x = \frac{y+z}{2}$

2.) $(2,4,-50,-61)$

3.) $b = y + 2$

4.) $\frac{3\pi}{16}$

5.) $a = 2$ or $a < \frac{3}{2}$

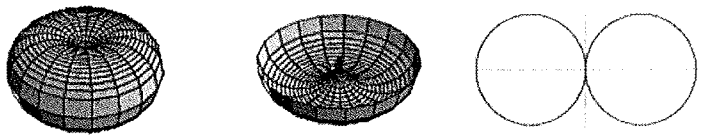
6.) $y = \frac{z+\sqrt{z^2+4}}{2}$

1.) Consider ΔABC with $\angle B \cong \angle C$, point D on $\overline{AB}$, point E on $\overline{AC}$, and point F on $\overline{BC}$. If $m\angle DFB = x^o$, $m\angle ADE = y^o$, and $m\angle FEC = z^o$, and ΔDEF is equilateral, find x in terms of y and z.

2.) The polynomial $f(x) = 4x^3 - 24x^2 - 2x + 7$ can be written in the form $f(x) = a(x-p)^3 + b(x-p) + c$ for real constants a, b, c, and p. Find the ordered quadruple (p, a, b, c).

3.) A bag contains 3 red marbles, b blue marbles, and y yellow marbles. It is known that the probability of randomly drawing a blue marble followed by a yellow marble without replacement is p. If an additional blue marble is added to the original bag before any marbles are drawn, the probability of randomly drawing a blue marble followed by a yellow marble without replacement is still p. Find b in terms of y.

4.) Consider a torus where a plane slicing vertically through the center shows tangent circular cross-sections, as shown to the right. If one of these circles has a diameter of R, consider also a sphere with a great circle cross-section having a radius of R. If V_T is the volume of the torus and V_S is the volume of the sphere, find the exact value of $\frac{V_T}{V_S}$.



5.) Find all real numbers a such that the equation $x + a = \sqrt{2x+3}$ has exactly one real solution for x.

6.) If $x < 0, y > 0$, and $z > 0$, solve for y in terms of z only given $z = x + 2y$ and $z^2 = x^2 + xy + y^2 - 3$.