

FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 4 Round 1
Arithmetic: Basic Statistics

1.) _____

2.) _____

3.) _____

1.) A sequence of ten consecutive prime numbers has a range of 27. Find the median of this sequence.

2.) The geometric mean of n numbers a_1, a_2, \dots, a_n is equal to the n th root of the product of the numbers: $\sqrt[n]{a_1 a_2 \dots a_n}$. A four-term arithmetic sequence has an average (arithmetic mean) of 10 and a range of 12. The geometric mean of the numbers of this sequence can be written as $p * \sqrt[4]{q}$ where p and q are integers greater than 1. Find pq .

↑
in simplest form

3.) A data set of 18 numbers has an average (arithmetic mean) of 70. Exactly k of these numbers ^{are} 40 (where $k > 0$). If all of the numbers with a value of 40 are dropped from the data set, the average of the remaining numbers increases by a whole number. Find the number of possible values of k .

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Match 4 Round 2

Algebra 1: Quadratic Equations

1.) _____

2.) _____

3.) _____

1.) Solve for all values of x : $x + \frac{12}{x} = 11 - x$.

2.) If $x^2 + (k + 2)x - k = 0$ has only one distinct real solution for x , find all possible values of k .

3.) Given the equation $px^2 + 3p^2 = p^3x + 3x$, $x = 6$ is one of 2 rational solutions for x . Find the other.

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Match 4 Round 3
Geometry: Similarity

1.) _____

2.) _____

3.) _____

1.) Consider triangle NES with right angle E . Point D lies on \overline{NE} and point Q lies on \overline{NS} such that $\overline{DQ} \parallel \overline{ES}$. If $NS = 20$, $NE = 12$, and $DE = 3.75$, find DQ .

2.) Consider regular hexagons $ABCDEF$ and $GHIJKL$. If $AD = 6$ and $GI = 10$, find the value of $\frac{\text{area } ABCDEF}{\text{area } GHIJKL}$.

3.) Consider trapezoid $FCML$ with bases \overline{FC} and \overline{ML} . The diagonals of the trapezoid intersect at point A . $FC = 6$ and the perpendicular distance from A to \overline{FC} is 4. If the area of $FCML$ is 36, find the height of the trapezoid.

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Match 4 Round 4
Algebra 2: Variation

1.) _____

2.) _____

3.) _____

- 1.) If y varies directly as the square of x and $y = 5$ when $x = 2$, find the positive value of x when $y = 80$.
- 2.) Assume that w varies jointly as x and the square root of y and inversely as the square of z . If x is reduced to one-third its value and z is reduced to one-half its value, y must be multiplied by c to ensure the value of w remains unchanged. Find the value of c .
- 3.) A sphere of solid Fairfieldium has a weight that varies as the cube of its diameter and a market value that varies as the square of its weight. A sphere 1.25 cm in diameter has a weight of $\frac{3}{16}$ oz., and a sphere of diameter 2.5 cm is worth \$4500. For a sphere of solid Fairfieldium with diameter of d cm, weight w oz, and value v dollars, find the ordered pair $\left(\frac{w}{d^3}, \frac{v}{w^2}\right)$.

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Match 4 Round 5 Precalculus:
Trigonometric Expressions and
DeMoivre's Theorem

1.) _____

2.) _____

3.) _____

1.) If $z = 1 + i$, find z^5 in rectangular ($a + bi$) form.

2.) For angles A and B in Quadrant I, if $\cos(A) = \frac{3}{5}$ and $\sin(B) = \frac{1}{3}$, find the value of $\cos(2A + B)$.

3.) If $\sec(4\theta) = \frac{A\sec^4(\theta)}{B+C\tan^2(\theta)+D\tan^4(\theta)}$, where $A, B, C,$ and D are relatively prime integers and $A > 0$, find the value of $A + B + C + D$.

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Match 4 Round 6

Miscellaneous: Conic Sections

1.) _____

2.) _____

3.) _____

1.) Find the radius of the circle with equation $x^2 + y^2 + 8x - 4y + k = 0$ if the circle contains the point $(-2, 6)$.

2.) For a parabola with the equation $x = ay^2 + by + c$, it is known that the distance from the vertex to the focus is equal to $|a|$, and that the focus has coordinates of $(\frac{7}{2}, 6)$ and lies to the left of the vertex. Find the ordered triple (a, b, c) .

3.) An ellipse with the equation $\frac{x^2}{k^2} + \frac{y^2}{10} = 1$ has foci that are exactly a units apart, where a is the length of the semi-major axis. Find all possible values of the length of the horizontal axis of the ellipse.

1.) _____

4.) _____

2.) _____

5.) _____

3.) _____

6.) _____

1.) Two poles of heights 5 feet and 12 feet stand vertically upward. A rope strung tightly from the top of one pole to the top of the other has a length of $7\sqrt{6}$ feet. A point P is found on the ground in between the poles such that the angles of elevation from P to the tops of each pole are complementary. A second rope is strung tightly from the top of one pole to P and then from P to the top of the other pole. This second rope has a total length in feet of $\sqrt{a} + b\sqrt{c}$ where a, b , and c are integers with a and c having no perfect square factors greater than 1. Find all possible ordered triples (a, b, c) .

2.) Assume y varies inversely as the n th power of x , where $n > 0$. If $y = 160$ when $x = 45$ and $y = 540$ when $x = 20$, find y when $x = 80$.

3.) An ellipse with the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ has the same foci as the hyperbola $9x^2 - 7y^2 = 63$. If the ellipse has area 6π and the area of the ellipse is found with πab , find the ordered pair (a, b) .

4.) The geometric mean of two numbers is found by taking the square root of the product of the numbers. For two given positive numbers a and b with $a > b$, the arithmetic mean is exactly three times the geometric mean. Find $\frac{a}{b}$ in simplest radical form.

5.) Consider function $f(x) = ax^2 + bx + c = a(x - h)^2 + k$ for nonzero a, b, c, h , and k . If $a > 0$, $h = k = 2c$, and $f(h + 1) = -a$, find all values p such that $f(p) = 0$.

6.) For how many natural numbers n , $2 \leq n \leq 100$, does one of the complex n th roots of $6 - 2\sqrt{3}i$ have an argument of $\frac{7\pi}{6}$?