FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 4 Round 1
Arithmetic: Basic Statistics

1.) 12

2.) 96

3.) 10

1.) A sequence of ten consecutive prime numbers has a range of 27. Find the median of this sequence.

2.) The geometric mean of n numbers $a_1, a_2, \ldots, a_n$ is equal to the nth root of the product of the numbers: $\sqrt[n]{a_1 a_2 \ldots a_n}$. A four-term arithmetic sequence has an average (arithmetic mean) of 10 and a range of 12. The geometric mean of the numbers of this sequence can be written as $p * \sqrt[4]{q}$ where p and q are integers greater than 1. Find pq.

↑ in simplest form

3.) A data set of 18 numbers has an average (arithmetic mean) of 70. Exactly k of these numbers ~~is~~ are 40 (where $k > 0$). If all of the numbers with a value of 40 are dropped from the data set, the average of the remaining numbers increases by a whole number. Find the number of possible values of k.

FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 4 Round 2
Algebra 1: Quadratic Equations

1.) $x = 4, x = \frac{3}{2}$

2.) $k = -4 \pm 2\sqrt{3}$

3.) $x = \frac{1}{4}$

1.) Solve for all values of : $x + \frac{12}{x} = 11 - x$.

2.) If $x^2 + (k+2)x - k = 0$ has only one distinct real solution for x, find all possible values of k.

3.) Given the equation $px^2 + 3p^2 = p^3x + 3x$, $x = 6$ is one of 2 rational solutions for x. Find the other.

FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 4 Round 3
Geometry: Similarity

1.) 11

2.) $\frac{27}{100}$

3.) $4\sqrt{3}$

1.) Consider triangle NES with right angle E. Point D lies on $\overline{NE}$ and point Q lies on $\overline{NS}$ such that $\overline{DQ}||\overline{ES}$. If $NS = 20$, $NE = 12$, and $DE = 3.75$, find DQ.

2.) Consider regular hexagons $ABCDEF$ and $GHIJKL$. If $AD = 6$ and $GI = 10$, find the value of $\frac{\text{area } ABCDEF}{\text{area } GHIJKL}$.

3.) Consider trapezoid $FCML$ with bases $\overline{FC}$ and $\overline{ML}$. The diagonals of the trapezoid intersect at point A. $FC = 6$ and the perpendicular distance from A to $\overline{FC}$ is 4. If the area of $FCML$ is 36, find the height of the trapezoid.

FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 4 Round 4
Algebra 2: Variation

1.) 8

2.) $c = \frac{9}{16}$

3.) $\left(\frac{12}{125}, 2000\right)$

1.) If y varies directly as the square of x and $y = 5$ when $x = 2$, find the positive value of x when $y = 80$.

2.) Assume that w varies jointly as x and the square root of y and inversely as the square of z. If x is reduced to one-third its value and z is reduced to one-half its value, y must be multiplied by c to ensure the value of w remains unchanged. Find the value of c.

3.) A sphere of solid Fairfieldium has a weight that varies as the cube of its diameter and a market value that varies as the square of its weight. A sphere 1.25 cm in diameter has a weight of $\frac{3}{16}$ oz., and a sphere of diameter 2.5 cm is worth \$4500. For a sphere of solid Fairfieldium with diameter of d cm, weight w oz, and value v dollars, find the ordered pair $\left(\frac{w}{d^3}, \frac{v}{w^2}\right)$.

FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 2 Round 5 Precalculus: Trigonometric Expressions & DeMoivre's Theorem

1.) $-4-4i$

2.) $\frac{-24-14\sqrt{2}}{75}$

3.) -3

1.) If $z = 1 + i$, find z^5 in rectangular $(a + bi)$ form.

2.) For angles A and B in Quadrant I, if $\cos(A) = \frac{3}{5}$ and $\sin(B) = \frac{1}{3}$, find the value of $\cos(2A + B)$.

3.) If $\sec(4\theta) = \frac{A\sec^4(\theta)}{B + C\tan^2(\theta) + D\tan^4(\theta)}$, where $A, B, C,$ and D are relatively prime integers and $A > 0$, find the value of $A + B + C + D$.

FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 4 Round 6
Miscellaneous: Conic Sections

1.) $2\sqrt{5}$

2.) $\left(-\frac{1}{2}, 6, -14\right)$

3.) $\frac{4\sqrt{30}}{3}, \sqrt{30}$

1.) Find the radius of the circle with equation $x^2 + y^2 + 8x - 4y + k = 0$ if the circle contains the point $(-2,6)$.

2.) For a parabola with the equation $x = ay^2 + by + c$, it is known that the distance from the vertex to the focus is equal to $|a|$, and that the focus has coordinates of $\left(\frac{7}{2}, 6\right)$ and lies to the left of the vertex. Find the ordered triple (a, b, c).

3.) An ellipse with the equation $\frac{x^2}{k^2} + \frac{y^2}{10} = 1$ has foci that are exactly a units apart, where a is the length of the semi-major axis. Find all possible values of the length of the horizontal axis of the ellipse.

Team Round

FAIRFIELD COUNTY MATH LEAGUE 2019-2020 Match 4 Team Round

1.) (70,4,14) (105,3,21)

2.) 67.5 or $\frac{135}{2}$

3.) $(3\sqrt{2}, \sqrt{2})$

4.) $17 + 12\sqrt{2}$

5.) $-1 + \sqrt{2}, -1 - \sqrt{2}$

6.) 8

1.) Two poles of heights 5 feet and 12 feet stand vertically upward. A rope strung tightly from the top of one pole to the top of the other has a length of $7\sqrt{6}$ feet. A point P is found on the ground in between the poles such that the angles of elevation from P to the tops of each pole are complementary. A second rope is strung tightly from the top of one pole to P and then from P to the top of the other pole. This second rope has a total length in feet of $\sqrt{a} + b\sqrt{c}$ where $a, b,$ and c are integers with a and c having no perfect square factors greater than 1. Find all possible ordered triples (a, b, c).

2.) Assume y varies inversely as the nth power of x, where $n > 0$. If $y = 160$ when $x = 45$ and $y = 540$ when $x = 20$, find y when $x = 80$.

3.) An ellipse with the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ has the same foci as the hyperbola $9x^2 - 7y^2 = 63$. If the ellipse has area 6π and the area of the ellipse is found with πab, find the ordered pair (a, b).

4.) The geometric mean of two numbers is found by taking the square root of the product of the numbers. For two given positive numbers a and b with $a > b$, the arithmetic mean is exactly three times the geometric mean. Find $\frac{a}{b}$ in simplest radical form.

5.) Consider function $f(x) = ax^2 + bx + c = a(x - h)^2 + k$ for nonzero $a, b, c, h,$ and k. If $a > 0, h = k = 2c$, and $f(h + 1) = -a$, find all values p such that $f(p) = 0$.

6.) For how many natural numbers n, $2 \leq n \leq 100$, does one of the complex nth roots of $6 - 2\sqrt{3}i$ have an argument of $\frac{7\pi}{6}$?