

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 4 Round 1
Arithmetic:
Basic Statistics

1.) 0.2

2.) 32

3.) $8\sqrt{2}$

1.) Let $S = \{\text{the 10 smallest natural numbers that are neither multiples of 3 nor multiples of 5}\}$. Find the positive difference between the arithmetic mean and the median of this set.

2.) A set of N Math SAT scores has arithmetic mean 550. When you remove four scores of 520, 540, 660, and 620, the arithmetic mean drops to 545. Find the value of N .

3.) The geometric mean of the numbers $x_1, x_2, x_3, \dots, x_n$ is defined to be $\sqrt[n]{x_1 x_2 x_3 \dots x_n}$. If T is the set of positive natural number factors of 128, what is the geometric mean of the elements of T ? Express your answer in simplest radical form.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 4 Round 2
Algebra 1:
Quadratic
Equations

1.) 0.5, 2.5

2.) $\frac{1}{2}, \frac{1}{5k-2}$

3.) 20, 24, 15

1.) Find all solutions to the equation $(x-3.5)^2=(2x-4)^2$

2.) If $k \neq 0.4$, give all solutions to the quadratic equation $(10k-4)x^2 - 5kx + 1 = 0$ in terms of k .

3.) Find all positive integer values of m such that the equation

$\frac{m}{4}x^2 - 25x + m = 0$ has two rational solutions that are between -10 and 10.

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Match 4 Round 4
Algebra 2:
Variation

1.) 0.75

2.) 12 mm

3.) $\frac{64}{27}$

1.) z varies inversely with the cube of $(y+2)$. If $z=6$ when $y=3$, what is the value of z when $y=8$?

2.) The resistance in ohms of a cylindrical electrical circuit element made from a certain material is directly proportional to its length and inversely proportional to its cross sectional area. If a resistor of length 6 mm has a radius of 4 mm and a resistance of 10 ohms, what is the length in mm of a resistor with resistance of 5 ohms and radius 8 mm?

3.) z varies jointly with the cube of x and the square of y , x varies directly with $v^{\frac{1}{2}}$ and y varies inversely with $w^{\frac{2}{3}}$. If $z=36$ when $v=9$ and $w=8$, what is w when $v=4$ and $z=54$?

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 4 Round 5
Trig Expressions

1.) -2

2.) $\sec^2 x - \csc^2 x$

3.) 32

1.) If $x = \frac{3\pi}{4}$, find the sum of $\cos(x) + \sin(x) + \cot(x) + \tan(x) + \sec(x) + \csc(x)$.

2.) If $0 < x < \frac{\pi}{2}$, express $(\tan(x) - \cot(x)) \left(\frac{\sec(x) + \csc(x)}{\cos(x) + \sin(x)} \right)$ in terms of powers of $\sec(x)$ and $\csc(x)$.

3.) What is the coefficient of $\cos^6(x)$ when $\cos(6x)$ is written as polynomial in $\cos(x)$?

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Match 4 Round 6
Conics

1.) $(\underline{1}, \underline{-2})$

2.) $\underline{\sqrt{5}}$

3.) $\underline{24}$

1.) The circle with equation $4x^2 - 8x + 4y^2 - ky = 380$ passes through the point $(7,6)$. Give the coordinates of the center of the circle.

2.) A parabola has its focus at $(0,1)$ and its directrix is the x -axis. The parabola passes through $(x,3)$ for some positive value of x . Find x . Express your answer in simplest radical form.

3.) The foci of the two hyperbolas $5x^2 - 20x - 4y^2 - 24y = 36$ and $7x^2 - 14x - 9y^2 - 72y = 74$ are the vertices of a quadrilateral. Find the area of this quadrilateral.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015 Match 4 Team Round

1.) 121.25

4.) 0, -1, $\frac{3}{4}$

2.) $\frac{15}{23}$

5.) $80\sqrt{10}$

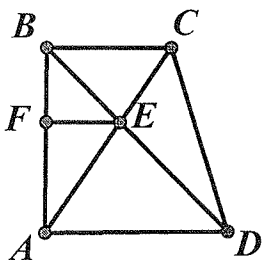
3.) $\frac{36}{7}$

6.) (4, 1) (7.75, 2.5)

1.) A variation function has form $f(x)=ax^b$ for some constants a and b . When $x=4$, $f(x)=2$. When $x=16$, $f(x)=16$. The interquartile range of a set is defined as the positive difference between its upper quartile (median of the upper half of the set) and lower quartile (median of the lower half of the set). Consider the set of ten numbers $\{f(1), f(4), f(9), f(16), \dots, f(100)\}$. Find the interquartile range of this set.

2. In $\triangle ABC$, the angle bisector of $\angle ACB$ intersects \overline{AB} at point D . If $AC=2y^2+16$, $CB=y+14$, $AD=y+16$, and $DB=12$, give the arithmetic mean of all possible values of y .

3. In trapezoid $ABCD$, the bases are \overline{AD} and \overline{BC} . \overline{AB} is perpendicular to both \overline{AD} and \overline{BC} . Diagonals \overline{BD} and \overline{AC} meet at E . A line segment is drawn from E to \overline{AB} parallel to \overline{AD} and intersects \overline{AB} at point F . If $AD=12$ and $BC=9$, find the length of \overline{EF} .



4. Find all values of $\sin(x)$ for which $\sin(3x) = \sin^2(x)$.

5. The eccentricity of an ellipse is this ratio: (distance of one of its foci from the center of the ellipse) to (length of semi-major axis). Kepler's third law of planetary motion states that the time it takes a planet to orbit the sun varies directly with some power of the length of the semi-major axis of its orbital path. The length of Venus' semi-major axis is 0.7 astronomical units, and the time it takes to orbit the sun in earth-years is the constant of variation multiplied by $\sqrt{0.343}$. Pluto has an eccentricity of 0.25 and the length of the semi-minor axis of its orbital path is $10\sqrt{15}$ astronomical units. The time it takes Pluto to orbit the sun in earth-years is the same constant of variation multiplied by what number? Express your answer in simplest radical form.

6. Give the coordinates of all points where the parabola $x = y^2 - y + 4$ intersects either of the asymptotes of the hyperbola $4x^2 + 8x - 25y^2 - 50y = 121$.