

FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 2 Round 1  
Arithmetic: Factors &  
Multiples

1.) 19

2.) 56

3.)  $p^8 + q^8$  or  $1 + p^8 q^8$

- 1.) How many integers less than 100 are the product of two odd (not necessarily distinct) primes?
- 2.) The greatest common factor of  $m$  and  $n$  is 24. The least common multiple of  $m$  and  $n$  is 360. Find the number of factors of  $mn$ .
- 3.) Let  $p$  and  $q$  be distinct prime numbers. If the greatest common factor of  $A$  and  $B$  is  $pq$  and the least common multiple of  $A$  and  $B$  is  $p^3q^3$ , find the value of  $\frac{A^4+B^4}{AB}$  in terms of  $p$  and  $q$ .

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Match 2 Round 2  
Algebra 1: Polynomials  
and Factoring

1.)  $\underline{\quad -6 \quad}$

2.)  $\underline{\quad 4 \quad}$

3.)  $\underline{(a + 4b - 2)(a + 2b + 2)}$

1.) If  $(x + 1)(x - 1) + x(x + 2)(x - 3)$  is written as  $ax^3 + bx^2 + cx + d$ ,  
find the value of  $a + b + c + d$ .

2.) For how many different values of  $k$  does  $x^2(x^k + 3x - 1) - x^4$  represent a  
quartic polynomial?

3.) Factor the following into the product of two trinomials with integer  
coefficients:  $a^2 + 6ab + 8b^2 + 4b - 4$ .

## FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 2 Round 3  
Geometry: Area & Perimeter

1.)  $\frac{121\pi}{8}$  cm<sup>2</sup>

2.) 1260 in<sup>2</sup>

3.) 32

1.) A circle is inscribed in a square with a diagonal length of 11 cm. What is the area of the circle in square centimeters?

2.) An isosceles trapezoid whose larger base is twice the length of its smaller base has a midsegment (median) length of 60 in and a perimeter of 178 in. What is its area in square inches?

3.) A rhombus has a perimeter of  $24\sqrt{5}$  units and an area of 76 square units. Find the sum of the lengths of the diagonals of the rhombus.

## FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 2 Round 4  
Algebra 2: Absolute  
Value & Inequalities

Remember to use AND or OR or the shorthand conjunction for a conjunction if you answer with  $<$ ,  $>$ ,  $\leq$ , or  $\geq$ . You may use union or intersection symbols if answering with interval notation.

1.)  $x = 6$  or  $x = \frac{4}{3}$

2.)  $(\frac{11}{4}, \frac{15}{4}, \frac{23}{4})$

3.)  ~~$x$~~   $(-\infty, \frac{1}{3}) \cup (\frac{1}{2}, \frac{11}{2}) \cup (\frac{17}{3}, \infty)$

1.) Solve for all values of  $x$ :  $|2x - 5| = x + 1$

2.) The compound inequality  $a < |x - b| < c$  has a solution set for  $x$  of  $(-2, 1) \cup (\frac{13}{2}, \frac{19}{2})$ . Write the ordered triple  $(a, b, c)$ .

3.) Solve for all values of  $x$ :  $\frac{1}{2|x-3|-5} < 3$ .

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Match 2 Round 5  
Precalculus: Law of Sines  
& Cosines

1.)  $\frac{-5}{28}$

2.)  $\frac{8}{5}$

3.)  $\frac{8}{3}$

1.) If for triangle  $ABC$ ,  $AB = 6$ ,  $BC = 7$ , and  $AC = 10$ , find the numerical value of  $\min(\cos(A), \cos(B), \cos(C))$ .

2.) Consider triangle  $ABC$  with point  $D$  on  $\overline{AC}$ . If  $AB = 6$ ,  $AD = 8$ , the area of triangle  $ABD$  is 10, and the area of triangle  $BCD$  is 2, find  $CD$ .

3.) Consider kite  $ABCD$  with  $AB = AD = x$  and  $BC = CD$ . If  $\tan(A) = \frac{3}{4}$ ,  $\angle A$  is supplementary to  $\angle C$ , and the perimeter of the kite can be written as  $kx$ , find the value of  $k$ .

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Match 2 Round 6  
Miscellaneous: Equations of  
Lines

1.)  $\frac{15}{2}$

2.)  $(10, -15)$

3.)  $y = -\frac{2}{3}x - \frac{13}{3}$

1.) If the line  $Ax + By = C$  is perpendicular to  $y = \frac{2}{3}x + 5$  but has the same  $y$ -intercept, find the value of  $\frac{AC}{B^2}$ .

2.) The line  $y = \frac{3}{5}x + 8$  can be represented parametrically by the equations  $y(t) = 6t - 1$  and  $x(t) = at + b$ . Find the ordered pair  $(a, b)$ .

3.) The point  $(5,1)$  lies on the circle  $(x - 1)^2 + (y + 5)^2 = 52$ . There are exactly two other points on the circle that are  $2\sqrt{26}$  units away from  $(5,1)$ . Find the equation of the line passing through these two points in slope-intercept form.

## Team Round

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1.) 32

4.)  $(x+y+z)(x-y-z)(x-y+z)(x+y-z)$

2.)  $\frac{48 + 25\sqrt{3}}{11}$

5.)  $\frac{75 + 25\sqrt{3}}{8}$

3.)  $a = -b - 6$

6.) 24

1.) The greatest common factor of  $M$  and 100 is 10. The least common multiple of  $M$  and 308 is 4620. Find the number of factors of the greatest possible value of  $M$ .

2.) Consider triangle  $ABC$  drawn on the Cartesian plane with  $AB = AC$  and with point  $A$  in Quadrant III. If point  $B$  is located at  $(0,5)$ , point  $C$  is located at  $(8,-1)$ , and the triangle has an area of  $25\sqrt{3}$  square units, find the slope of the line  $\overleftrightarrow{AB}$  in simplest radical form.

3.) If  $a$  and  $b$  are real numbers such that  $a < 0 < b$  and the equation  $|x - a| = |x - b|$  shares a solution for  $x$  with the equation  $|x - b| = b + 3$ , find  $a$  in terms of  $b$ .

4.) Factor into trinomials with no power higher than 1:  $(x^2 - y^2 - z^2)^2 - 4y^2z^2$

5.) A cruise ship is located 20 km away from a Coast Guard outpost at a bearing of 20 degrees East of North. The cruise ship is moving at a speed of 30 km/hr at a bearing of 10 degrees North of East. If a speedboat leaves the outpost to intercept the cruise ship's course without changing direction and travels at a rate of 50 km/hr, find the total distance in kilometers traveled by the speedboat by the time it intercepts the cruise ship.

6.) Consider trapezoid  $TRAP$  with  $\overline{TR} \parallel \overline{AP}$  and  $m\angle A > m\angle P$ . The diagonals intersect at point  $D$ . If  $TR = 6$  and the perpendicular distance from  $D$  to  $\overline{AP}$  is 8, find the area of triangle  $TPD$ .