FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 1 Round 1
Arithmetic: Percents

1)__N=____1000__________

2.) __$____500________________

3.) C=__252__ R=___120__

1.) 30% of 40% of 50% of N is 60. Find N.

$$(0.3)(0.4)(0.5)N = 60$$

$$0.06N = 60$$

$$N = \frac{60}{0.06} = 1000$$

2) The price of an item was decreased by 10%, then increased by \$20, then decreased by 30%, then increased by \$51, then decreased by 40% If the final price was \$228, what was the original price?

$$Let\_N = original\_price$$

$$0.6(0.7(0.9N + 20) + 51) = 228$$

$$0.7(0.9N + 20) + 51 = \frac{228}{0.6} = 380$$

$$0.7(0.9N + 20) = 380 - 51 = 329$$

$$0.9N + 20 = \frac{329}{0.7} = 470$$

$$0.9N = 470 - 20 = 450$$

$$N = \frac{450}{0.9} = 500$$

3.) In Buford's exercise program, he burns 5% more calories per minute when he cycles than when he runs. He cycles for half an hour and burns C calories. He runs for 15 minutes and burns R calories. Three more than twice the total number of minutes Buford exercised is numerically 25% of the total number of calories burned. Find C and R.

$$\frac{C}{30} = 1.05(\frac{R}{15})$$

$$\frac{C}{30} = 0.07R$$

$$C = 2.1R$$

$$3 + 2 * 45 = 93$$

$$93 = 0.25(C + R)$$

$$93 = 0.25(2.1R + R)$$

$$93 = 0.25(3.1R)$$

$$372 = 3.1R$$

$$R = 120,$$

$$C = 2.1(120) = 252$$

FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 1 Round 2
Arithmetic: Equations

1)____x=___ $\frac{119}{39}$ _____

2.) __________9________

3.) __y=____ $6, \frac{1}{7}$ __________

1) Solve for x: $6x-5(x-4(x-3(x-2)))=1$

3(x-2)=3x-6. x-(3x-2)=x-(3x-6) = -2x+6. 4(x-(3x-2)) = 4(-2x+6) = -8x+24. x-4(x-3(x-2)) = x-(-8x+24) = 9x-24. 6x-5(x-4(x-3(x-2)))=6x-5(9x-24) = 6x-45x+120 = -39x+120. If

-39x+120=1, -39x=-119, x = $\frac{119}{39}$

2) m and n are natural numbers such that $1 \le m \le 100$ and $1 \le n \le 100$. How many distinct ordered pairs (m,n) solve the equation $m^2 - 4n = 1$?

m and n are natural numbers such that $1 \le m \le 100$ and $1 \le n \le 100$. How many distinct ordered pairs (m,n) solve the equation $m^2 - 4n = 1$?

4n+1 is odd. No even numbers have perfect squares that are odd. All odd perfect squares are of the form 4n+1 for some n, since if

$m=2p+1$, then $m^2 = 4p^2 + 4p + 1 = 4(p^2+p)+1$. If $m=1$, $n=0$,which is not in the given domain. We have (m,n) belongs to {(3,2),(5,6),(7,12),(9,20),(11,30),(13,42),(15,56),(17,70),(19,90), and then n becomes greater than 100, so there are 9 such ordered pairs.

3) Solve for y: $(y+1)(y+2)(2y-3)=2y(y+8)(y-3)$.

$(y+1)(y+2)(2y-3)= (y^2 + 3y+2)(2y-3)=2y^3 + 3y^2 - 5y - 6$

$2y(y+8)(y-3) = 2y(y^2 +5y-24)= 2y^3 + 10y^2 - 48y$.

$2y^3$ terms can be subtracted from each side, so

$3y^2 - 5y - 6 = 10y^2 - 48y$

$7y^2 - 43y+6=0$

$(y-6)(7y-1)=0$

$$y=6, y=\frac{1}{7}$$

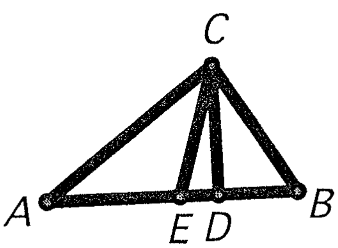
FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 1 Round 3
Geometry: Triangles
And Quadrilaterals

1.) $\frac{7}{10}$ cm

2.) 65 cm

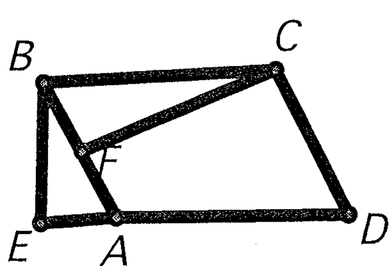
3.) $\frac{12+28\sqrt{3}}{3}$ cm



1.: In right ΔABC of sides 3", 4", and 5". the right angle is at C. An altitude is drawn from C and meets $\overline{AB}$ at D. A median is drawn from C and meets $\overline{AB}$ at E. Find the distance DE.

AE=BE=2.5" by definition of median. The length of altitude $\overline{CD}$ is $\frac{3*4}{5}=\frac{12}{5}$. Assume CB=3. Then $(BD)^2=(CB)^2-(CD)^2 = 3^2-(\frac{12}{5})^2=\frac{225}{25}-\frac{144}{25}=\frac{81}{25}$, so BD = $\frac{9}{5}$. DE =

$$\frac{5}{2}-\frac{9}{5}=\frac{25-18}{10}=\frac{7}{10}$$



2. In the figure above, ABCD is a parallelogram. A lies on $\overleftrightarrow{DE}$, F lies on $\overleftrightarrow{BA}$, $\overline{BE}\perp\overline{AE}$, and $\overline{BF}\perp\overline{CF}$. BE = 20 cm and CF =16 cm. The area of ABCD is 200 cm^2. Find the perimeter of ABCD.

BE= height of the parallelogram = 10 cm, therefore AD=BC=20 cm to give area 200 cm^2. $\angle DAB$ is supplementary to both $\angle BAE$ and $\angle FBC$, so $\angle BAE = \angle FBC$. ΔBCF and ΔABE are both right triangles with one other equal angle, so they are similar to each other. So $\frac{BE}{CF} = \frac{AB}{BC}$, $\frac{10}{16} = \frac{AB}{20}$, $AB = 12.5$. AB=CD, so the perimeter is 20 + 12.5 + 20 + 12.5 = 65 cm.

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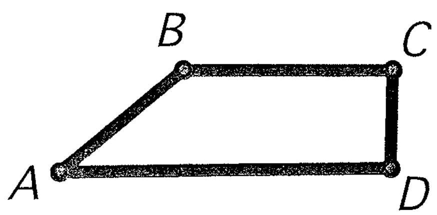


3. In trapezoid ABCD above, AB=BC, $\overline{AD}$ is parallel to $\overline{BC}$, $\angle B = 120°$, $\angle C = \angle D = 90°$, CD=4 cm Find the perimeter of the trapezoid.

Let AB=BC=x. Draw a line from B parallel to $\overline{CD}$ intersecting $\overline{AD}$ at E. Then ΔADE is a 30-60-90 triangle with the longer leg $\overline{BE}$ measuring 4 cm. Then AE = $\frac{4}{\sqrt{3}}$ cm, and AB = $\frac{8}{\sqrt{3}}$ cm. Therefore, BC= $\frac{8}{\sqrt{3}}$ cm, and AD = $\frac{8}{\sqrt{3}} + \frac{4}{\sqrt{3}} = \frac{12}{\sqrt{3}}$ cm. Total perimeter is $\frac{8}{\sqrt{3}} + 4 + \frac{12}{\sqrt{3}} + \frac{8}{\sqrt{3}} = 4 + \frac{28}{\sqrt{3}} = \frac{12 + 28\sqrt{3}}{3}$

FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 1 Round 4
Algebra 2
Simultaneous
Equations

1.)___x=24, y= -9______________________________

2.)___ ____(0.6, 0.8), (1,0)________________________

3.)________ $x=-2, y=\frac{2}{5}$ ________________________

1.) Solve for x and y:

$$\frac{x}{4}-\frac{y}{3}=9$$

$$y=\frac{x}{6}-13$$

$$\frac{x}{4} - \frac{y}{3} = 9$$

$$y = \frac{x}{6} - 13$$

$$\frac{x}{4} - \frac{(\frac{x}{6} - 13)}{3} = 9$$

$$\frac{x}{4} - \frac{x}{18} + \frac{13}{3} = 9$$

$$\frac{9x}{36} - \frac{2x}{36} = 9 - \frac{13}{3}$$

$$\frac{7x}{36} = \frac{14}{3}$$

$$21x = 14 * 36$$

$$7x = 14 * 12 = 168$$

$$x = 24$$

$$y = \frac{24}{6} - 13 = -9$$

2. Solve for all ordered pairs (a,b)

$$2a + b = 2$$

$$a^2 + b^2 = 1$$

$$b = -2a + 2$$
$$a^2 + (-2a + 2)^2 = 1$$
$$a^2 + 4a^2 - 8a + 4 = 1$$
$$5a^2 - 8a + 3 = 0$$
$$(5a - 3)(a - 1) = 0$$
$$a = 0.6, a = 1$$
$$If\_a = 0.6, b = 2 - (2(0.6)) = 0.8$$
$$If\_a = 1, b = 0$$

3. $\frac{xy}{x+y} = \frac{1}{2}$, so $\frac{x+y}{xy} = 2$. $\frac{xy}{x-y} = \frac{1}{3}$, so $\frac{x-y}{xy} = 3$.

These can be broken up into $\frac{1}{x} + \frac{1}{y} = 2$ and $\frac{1}{y} - \frac{1}{x} = 3$. Add the equations to get $\frac{2}{y} = 5$, so $y = \frac{2}{5}$. Then $\frac{1}{\frac{2}{5}} - \frac{1}{x} = 3$, so $\frac{5}{2} - \frac{1}{x} = 3$, $-\frac{1}{x} = \frac{1}{2}$, so $x = -2$

FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 1 Round 5
Trig
Right Triangles

1.) ____________ $\frac{13}{27}$ __________

Diagrams are not necessarily drawn to scale

2.) ____________ $\frac{5}{2}$ ______ft/sec

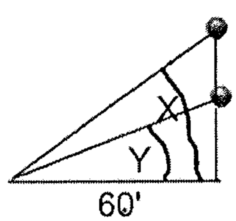
3.) __________15____________

1. In right triangle ABC, the right angle is at C. $BC = \frac{4}{9}, \sin(B) = \frac{5}{13}$. Find AB.

$$\cos(\angle B) = \sqrt{1 - (\frac{5}{13})^2} = \frac{12}{13}.$$

$$\frac{12}{13} = \frac{\frac{4}{9}}{AB}, 12 * AB = 13 * (\frac{4}{9}),$$

$$AB = \frac{13*4}{12*9} = \frac{13}{3*9} = \frac{13}{27}$$



2. A balloon is rising vertically. You are standing 60 feet away from the balloon horizontally. At a given time, the angle of elevation to the balloon is Y. Three seconds later,

the angle of elevation to the balloon is X.

$\sin(X)=\frac{5}{13}, \sin(Y)=\frac{7}{25}$

What is the average rate of change in height of the balloon during the three second interval in feet per second?

$\tan(X)=\frac{5}{12}, \tan(Y)=\frac{7}{24}$

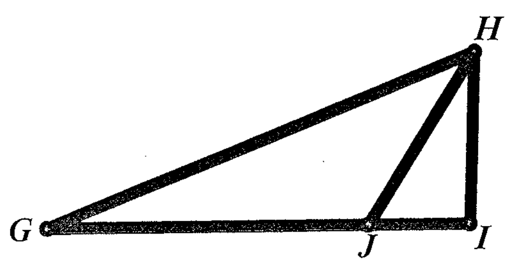
Height of balloon at initial time is P, where

$\frac{P}{60}=\frac{7}{24}, P=\frac{420}{24}=\frac{35}{2}$

Height of balloon three seconds later is Q, where

$\frac{Q}{60}=\frac{5}{12}, Q=\frac{300}{12}=25$

Difference in heights is $25-\frac{35}{2}=\frac{15}{2}$ Average rate of change of height is $\frac{\frac{15}{2}}{3}=\frac{15}{6}=\frac{5}{2}$ feet per second



3. In the diagram above, HI=4, JI=1, $\cos(\angle HGI)=4\cos(\angle HJI)$ and $\angle I$ is a right angle.
   Find GJ.

$$JH = \sqrt{1^2 + 4^2} = \sqrt{17}$$

$$\cos(\angle HJI) = \frac{1}{\sqrt{17}}$$

$$\cos(\angle HGI) = \frac{4}{\sqrt{17}} = \frac{GJ+1}{\sqrt{(GJ+1)^2+16}}, so$$

$$\sqrt{17}(GJ+1) = 4\sqrt{(GJ+1)^2+16} = 4\sqrt{(GJ^2+2*GJ+17}$$

$$17((GJ)^2+2*GJ+1) = 16((GJ)^2+2*GJ+17)$$

$$17((GJ)^2+34*GJ+17 = 16((GJ)^2+32*GJ+272$$

$$(GJ)^2+2*(GJ)-255 = 0$$

$$(GJ-15)(GJ+17) = 0$$

$$GJ \neq -17, so$$

$$GJ = 15$$

FAIRFIELD COUNTY MATH LEAGUE 2019-2020

Match 1 Round 6
Coordinate Geometry

1)____ $y=\frac{-3}{4}x+\frac{5}{2}$ ________________

2)________12 ______________________

3) ______-4,0,2,6_________________

1.) Give the equation of the line parallel to 3x+4y=24 passing through the point (6,-2). Express your answer as y=mx+b for constants m and b

3x+4y=24 can be transformed to

$$4y=-3x+12$$

$$y=\frac{-3}{4}x+3$$

so the desired slope is $\frac{-3}{4}$.

$$y+2=\frac{-3}{4}(x-6)$$

$$y+2=\frac{-3}{4}x+\frac{9}{2}$$

$$y=\frac{-3}{4}x+\frac{5}{2}$$

Or; 3*6+4(-2)=10, so 3x+4y=10, and then solve for y.

2.) How many distinct ordered pairs (a,b) exist such that a and b are both integers, and the length of the segment connecting (a,2) and (3,b) is 5 ?

We must have $\sqrt{(a-3)^2+(2-b)^2}=5$. In order for a and b to be both integers, one set of possibilities is $a-3=\pm4\_and\_2-b=\pm3$ or $a-3=\pm3\_and\_2-b=\pm4$. For each of the four possibilities for a-3, there are 2 possibilities for 2-b. Therefore, there are 8 distinct ordered pairs: (7,-1), (7,5),(-1,-1), (-1,5), (6, -2), (6,6),(0,-2), (0,6).
It is also possible that (a-2)=0 and (3-b)=±5, or, (3-b)=0 and (a-2)= ±5. This gives 4 more possibilities: (2,-2), (2,-8), (7,3),and (-3,3). so altogether there are 12 distinct ordered pairs.

3) $\overline{AB}$ has length $2\sqrt{5}$. The perpendicular bisector of $\overline{AB}$ is the line y= -2x+13. The distance from (1,11) to the midpoint of $\overline{AB}$ is $3\sqrt{5}$. Give the four possible values for the x-coordinate of point A.

$\overline{AB}$ must have slope $\frac{1}{2}$. If it has length $2\sqrt{5}$, its midpoint is two units right and one unit up from one of the endpoints, since $\sqrt{2^2+1^2}=\sqrt{5}$
In order for a point on y= -2x+13 to have distance from (1,11) to the midpoint of $\overline{AB}$ be $3\sqrt{5}$, it must be three units right and six units down from (1,11) or three units left and six units up from (1,11) since $\sqrt{3^2+(-6)^2}=3\sqrt{5}$ and $\frac{-6}{3}=-2$. The two possible midpoints are then (4,5) and (-2,17).

$\overline{AB}$ must have slope $\frac{1}{2}$. If it has length $2\sqrt{5}$, its midpoint is two units right and one unit up from one of the endpoints, since $\sqrt{2^2+1^2}=\sqrt{5}$
If the midpoint is (4,5), the two endpoints are (6,6) and (2,4). If the midpoint is (-2,17), the two endpoints are (-4,16) and (0,18). The possible x-coordinates of point A are -4, 0, 2, 6.

FAIRFIELD COUNTY MATH LEAGUE 2019-2020 Team Round Match 1

1.)_M=__20__N=__40__P=__80__

2.) $\frac{3-\sqrt{3}}{2}$

3.) 45

4.) $\frac{-5}{3}, -1$

5.) 32

6.) (-3,1), (6,4),(6,-2)

1.)_ M≠N≠P≠0. M% of P is 24 less than N. N% of M is one-tenth of P. P% of N is eight-fifths of M. Find M, N, and P.

$$\frac{M}{100}*P=N-24, \frac{N}{100}*M=\frac{1}{10}P, \frac{P}{100}*N=\frac{8}{5}M$$

From the second equation, MN=10P, substitute $P=\frac{MN}{10}$ into the other equations. From the third equation,

$$\frac{MN}{1000}*N=\frac{8}{5}M$$

$$N^2=\frac{8000}{5}=1600, N=40$$

Therefore $\frac{P}{100}*40=\frac{8}{5}M$ and $\frac{M}{100}*P=40-24$.

$$\frac{40P}{100}=\frac{8}{5}M, 200P=800M, P=4M.$$

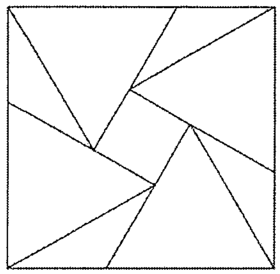
$$Substitute\_into\_\frac{M}{100}P=16,$$

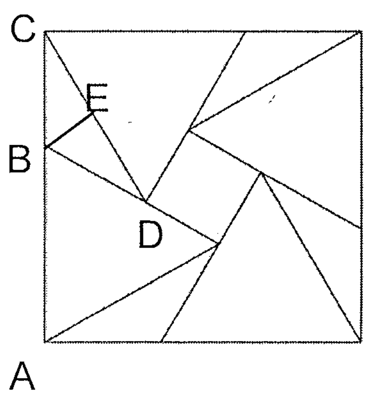
$$MP=1600$$

$$M(4M)=1600$$

$$M^2=400, M=20$$

$$M=20, N=40, P=80$$

2.  A square of side 1 cm has four congruent equilateral triangles constructed in it as shown. Find the largest side of any of the triangles in the diagram.



Let AB = x, then BC=1-x. CD is also x. ΔCBD is isosceles with angles of 30, 30, and 120 degrees. Draw altitude $\overline{BE}$ to make two 30-60-90 triangles where ED=CE=(1/2)x

Then in ΔEBC, EB is the smaller leg and is $\frac{1}{\sqrt{3}}$ times the longer leg, so EB= $\frac{x}{2\sqrt{3}}$.

BC is the hypotenuse and is twice the shorter leg, so BC = $\frac{2x}{2\sqrt{3}} = \frac{x}{\sqrt{3}}$. Now AB+BC=1, so

$$x + \frac{x}{\sqrt{3}} = 1$$

$$x\sqrt{3} + x = \sqrt{3}$$

$$x(1+\sqrt{3}) = \sqrt{3}$$

$$x = \frac{\sqrt{3}}{1+\sqrt{3}} = \frac{\sqrt{3}(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} = \frac{\sqrt{3}-3}{-2} = \frac{3-\sqrt{3}}{2}$$

3.) Find the area of the quadrilateral whose vertices are the pairwise intersections of the lines

$$y=\frac{1}{2}x-3, y=-2x+12, x-2y=-9,\_and\_8x-y=-12$$

The first and third lines have the same slope, so this is a trapezoid. There are right angles where the first and third lines intersect the second line, so the height of the trapezoid is the length of the segment connecting the intersections of the second line with the first and third. Find all the points of intersection and find the area of the trapezoid. One intersection point is found by

$$\frac{1}{2}x-3=-2x+12, \frac{5}{2}x=15, x=6$$

$$y=\frac{1}{2}(6)-3=0$$

$$(6,0)$$

The third line can be written $y=\frac{1}{2}x+\frac{9}{2}$. The second and third lines intersect where

$$\frac{1}{2}x+\frac{9}{2}=-2x+12$$

$$\frac{5}{2}x=\frac{15}{2}$$

$$x=3, y=-2(3)+12=6$$

$$(3,6)$$

The fourth line is y=8x+12, and intersects and the first and third where is

$$8x+12=\frac{1}{2}x-3$$

$$\frac{15}{2}x=-15, x=-2, y=8(-2)+12$$

$(-2,-4)$

and

$$8x+12=\frac{1}{2}x+\frac{9}{2}$$

$$\frac{15}{2}x=-\frac{15}{2}, x=-1, y=8(-1)+12=4$$

$(-1,4)$

The height of the trapezoid is $\sqrt{(6-3)^2+(0-6)^2}=3\sqrt{5}$

The bases of the trapezoid have lengths

$\sqrt{(6-(-2))^2+(0-(-4))^2}=\sqrt{64+16}=4\sqrt{5}$ and

$\sqrt{(6-4)^2+(3-(-1)^2}=\sqrt{4+16}=2\sqrt{5}$.

Area of the trapezoid is

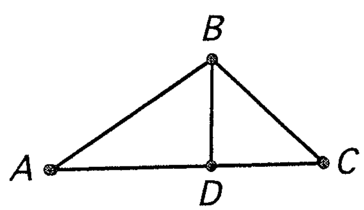
$$\frac{1}{2}(3\sqrt{5})(2\sqrt{5}+4\sqrt{5})$$

$$=\frac{1}{2}(3\sqrt{5})(6\sqrt{5})$$

$$=\frac{1}{2}*3*6*5=45$$

4.) Solve for x: $x^2(x-1)(3x+1)-7x+11=(((3x-2)x+2)x+1)x+16$

$$x^2(x-1)(3x+1)-7x+11=(((3x-2)x+2)x+1)x+16$$
$$x^2(3x^2-2x-1)-7x+11=((3x^2-2x)+2)x+1)x+16$$
$$3x^4-2x^3-x^2-7x+11=(3x^3-2x^2+2x+1)x+16$$
$$3x^4-2x^3-x^2-7x+11=3x^4-2x^3+2x^2+x+16$$
$$-x^2-7x+11=2x^2+x+16$$
$$3x^2+8x+5=0$$
$$(3x+5)(x+1)=0$$
$$x=\frac{-5}{3}, x=-1$$



5.)

Altitude $\overline{BD}$ is drawn for ΔABC. A and C are acute angles.

$$\cos(\angle A)=\frac{4}{5}, \cos(\angle C)=\frac{12}{13}$$

BD = 4 cm. Find the perimeter of ΔABC

$$\cos(\angle A) = \frac{4}{5}, \cos(\angle C) = \frac{12}{13}$$

$$\sin(\angle C) = \frac{5}{13}, \frac{5}{13} = \frac{4}{BC}, BC = \frac{52}{5}$$

$$\sin(\angle A) = \frac{3}{5}, \frac{3}{5} = \frac{4}{AB}, AB = \frac{20}{3}$$

$$\tan(\angle A) = \frac{5}{12}, \frac{5}{12} = \frac{4}{AD}, AD = \frac{48}{5}$$

$$\tan(\angle C) = \frac{3}{4}, \frac{3}{4} = \frac{4}{CD}, CD = \frac{16}{3}$$

$$Total\_perimeter\_is$$

$$\frac{52}{5} + \frac{20}{3} + \frac{48}{5} + \frac{16}{3} = \frac{100}{5} + \frac{36}{3} = 20 + 12 = 32$$

6.) Give all ordered pair solutions (x,y) for the system

$$x^2 - 4x + (y-1)^2 = 21$$

$$x - 2 = y^2 - 2y - 4$$

Complete the square on the first equation to get

$$x^2 - 4x + 4 + (y-1)^2 = 25$$

$$(x-2)^2 + (y-1)^2 = 25$$

Complete the square on the second equation to get

$$x - 2 = (y-1)^2 - 5$$

$$(y-1)^2 = x + 3, so$$

$$x^2 - 4x + 4 + x + 3 = 25$$

$$x^2 - 3x + 7 = 25$$

$$x^2 - 3x - 18 = 0$$

$$(x-6)(x+3) = 0$$

$$x = 6\_or\_x = -3$$

$$If\_x = 6,$$

$$6 + 3 = (y-1)^2, (y-1)^2 = 9,$$

$$y - 1 = \pm 3, y = 4\_or\_y = -2$$

$$If\_x = -3,$$

$$-3 + 3 = (y-1)^2, 0 = (y-1)^2$$

$$y = 1$$

$$(6,4), (6,-2), (-3,1)$$