FAIRFIELD COUNTY MATH LEAGUE 2017-2018

Match 3 Round 1
Arithmetic: Scientific Notation and Bases

1.) __A=1_____B=4__________

2.) _K A U Y I K E A T I_____

3.) _ 2, -1, 0, 1, 2_________

1.)_ ABB_5 is one less than one half of ABB_8. Find A and B.

25A + 5B + B = 0.5(64A + 8B + B) -1
25A + 6B = 32A + 4.5B -1
50A + 12B = 64A + 9B – 2
14A – 3B = 2
Since A and B must be digits in base 5, they must be 0,1,2,3,or 4. The only combination that works is A = 1, B = 4

2.)_ Mr. Matte from Greens Farms was on the TV show "Superhuman" last summer and solved a problem by converting a sequence of ice cream flavors to base 3 numbers and then converting ten base 3 numbers into letters using A=1, B=2,… Z=26. If the ten base 3 numbers were in this order: $102_3, 001_3, 210_3, 221_3, 100_3, 102_3, 012_3, 001_3, 202_3, 100_3$ write the 10 letter pattern that he needed to remember.

1=A, 2=B, 3=C, 4=D, 5=E, 6=F, 7=G, 8=H, 9=I, 10=J, 11=K, 12=L, 13=M
14=N, 15=0, 16=P, 17=Q, 18=R, 19=S, 20=T, 21=U, 22=V, 23=W, 24=X, 25=Y, 26=Z.

102=1*3^2+0*3^1+2*3^0=11, so K
001=-0*3^2+0*3^1+1*3^0=1, so A
210=2*3^2+1*3^1+0*3^0=21, so U
221=2*3^2+2*3^1+1*3^0=25, so Y
100=1*3^2+0*3^1+0*3^0=9, so I
102=1*3^2+0*3^1+2*3^0=11, so K
012=0*3^2+1*3^1+2*3^0=5, so E
001=0*3^2+0*3^1+1*3^0=1, so A

202=2*3^2+0*3^1+2*3^0=20, so T
100=1*3^2+0*3^1+0*3^0=9, so I

3.) Find all integers n such that $\frac{(3*10^n)^{2n}(5*10^n)^3}{(1.5*10^n)^n(5*10^n)^{n+2}}$ is between 0.000001 and 1000000.

$$\frac{(3*10^n)^{2n}(5*10^n)^3}{(6*10^n)^n(5*10^n)^{n+2}} =$$

$$\frac{3^{2n}*10^{2n^2}*5^3*10^{3n}}{6^n*10^{n^2}*5^n*5^2*10^{n^2}}$$

$$= \frac{9^n*5*10^{3n}}{6^n*5^n} = 0.3^n*5*10^{3n}$$

$$= 5*300^n$$

True for n=0, since expression is 5. If n=1, expression is 1500, OK. n=2, $5*300^2$ <1000000., but for n=3, $5*300^3$ >10000000

If n=-1, $\frac{5}{300} > 0.000001$

If n=-2, $\frac{5}{300^2} = \frac{5}{90000} > 0.000001$

If n=-3,

$$\frac{5}{300^3} = \frac{5}{27*10^6} = \frac{5}{27}*\frac{1}{10^6} < 0.000001$$

So the integers are -2, -1, 0, 1, 2

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Match 3 Round 2
Algebra: Word Problems

1.) _________67________________

2.) __M= 15 ml________N= 6 ml______

3.) ________________1.5____________mph

1.) The smallest of a set of N consecutive integers is -32 and the sum of the integers is 67. How many integers are in the set?

Winning question in the National MathCounts competition in 2014.
From -32 to 32 adds to 0, which is 65 integers. 33 and 34 then add to 67, so there are 67 integers total.

2.) You have M milliliters of a solution of water and acid that is 30% acid. If you add N milliliters of pure acid, you get a solution that is 50% acid. If you then add 21 milliliters of pure acid, you get a solution that is 75% acid. Find M and N.

Acid content is originally 0.3M. Add N ml of acid so
$0.3M + N = 0.5(M+N)$, so $0.3M + N = 0.5M + 0.5N$, so $0.5N=0.2M$, and $M=2.5N$. Then add an additional 21 ml of acid so that
$0.3M + N +21 = 0.75(M+N+21)$. Substitute 2.5N for M.
$0.3(2.5N) + N + 21 = 0.75(2.5N+N+21)$
$0.75N+N+21 = 2.625N+15.75$
$0.875N=5.25$
$N=6$, $M=2.5*6=15$.

3) A person in a paddleboat travels along a river and on a lake. On the river, she paddles 2 miles with a current of 1.5 mph in the same direction as she is traveling. On the lake she paddles 0.5 miles with no current. The total time for the trip is one hour. If she paddles at a constant speed , what is her paddling speed in mph?

Let x=paddling speed
x+1.5 = speed down river
Time = Distance/Speed

$$\frac{2}{x+1.5}+\frac{0.5}{x}=1$$

$$x^2-x-0.75=0$$

$$4x^2-4x-3=0$$

$$(2x-3)(2x+1)=0$$

$$x=1.5\_or\_x=-0.5$$

$$x=1.5\_mph$$

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Match 3 Round 3
Geometry: Polygons

1.) _________72____________

2.) _________104____________

3.) _______25, 27, 32_______

1.)The measure of one interior angle of a regular polygon is 175 degrees. How many sides does the polygon have?

$$175 = \frac{180(n-2)}{n}, 175n = 180n - 360, 5n = 360, n = 72$$

2) The number of sides plus the number of diagonals of a regular polygon add up to 120. Find the number of diagonals of the polygon.

$$n + \frac{n(n-3)}{2} = 120$$

$$2n + n^2 - 3n = 240$$

$$n^2 - n - 240 = 0$$

$$(n-16)(n+15) = 0$$

$$n = 16$$

Number of diagonals is $\frac{16(16-3)}{2} = 104$

3) Two regular polygons of sides M and N respectively where M≠N are such that their exterior angles are both whole numbers of degrees and the exterior angles add up to 60 degrees. Give all possible values of M+N.

We need distinct factors of 360 that add up to 60 (distinct, so not 30 and 30) The only possibilities are 15 and 45, 20 and 40, and 24 and 36. Using the formula $\frac{360}{N}$, 15 and 45 correspond to sides 24 and 8, so one possibility is 32; 20 and 40 correspond to 9 and 18, so one possibility is 27; 24 and 36 correspond to 15 and 10 so one possibility is 25. Answer is 25, 27, 32.

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Match 3 Round 4 Algebra 2: Functions and Inverses

1.) _ 36x + 11_____

Note: The inverse of a function is not necessarily itself a function.

2). _____$\left(-\infty,\frac{2}{3}\right)\cup\left(\frac{2}{3},2\right)\cup(2,\infty)$___

3.) _ Domain:__ $(-\infty,-3]$__ Range: _____ $(-\infty,-\frac{4}{3}]$___

1.) If $f(x)=3x+2$ and $g(x)=4x-5$, find $f(g(f(x)))$.

$$g(f(x))=4(3x+2)-5=12x+8-5=12x+3$$

$$f(12x+3)=3(12x+3)+2=36x+11$$

2.) What is the range of $y=\dfrac{2x^2-6x+4}{x^2-5x+4}$?

$y=\dfrac{2(x-1)(x-2)}{(x-1)(x-4)}$ so there is a removable discontinuity at x=1. When x=1, $\dfrac{2(x-2)}{x-4}=\dfrac{2}{3}$, so y cannot be $\dfrac{2}{3}$. There is a vertical asymptote at x=4, and y becomes infinitely large positive on one side and infinitely large negative on the other. There is a horizontal asymptote at y=2 and k(x) does not cross this at any point , so the range is all reals except for 1 and $\dfrac{2}{3}$.

3.) Let $h(x)=\sqrt{x-4}+3$, and let $g(x)=-h(-3x)$. Find the domain and range of g^{-1}.

If you choose to use inequalities instead of interval notation, use x for the domain and y for the range.

$$h(-3x) = \sqrt{(-3x)-4} + 3 = \sqrt{-3x-4} + 3$$

$$-h(-3x) = -\sqrt{-3x-4} - 3$$

Find its inverse by solving $x = -\sqrt{-3y-4} - 3$ for y.

$$x = -\sqrt{-3y-4} - 3$$

$$x + 3 = -\sqrt{-3y-4}$$

$$x + 3 = \sqrt{-3y-4}$$

$9 + 6x + x^2 = -3y - 4$ but be careful here! Since you are squaring both sides, part of this will be extraneous!

y must be less than or equal to $\frac{-4}{3}$ and 3+x $\geq$0, so x$\leq$-3.

$y = -\frac{1}{3}x^2 - 2x - \frac{13}{3}$. Only x values where x$\leq$--3 are acceptable. The vertex of the parabola is at x= $\frac{-b}{2a} = \frac{2}{2(\frac{-1}{3})} = -3$, y=$\frac{-4}{3}$. Take only the left side of the parabola, so $\begin{matrix} x \leq -3 \\ y \leq \frac{-4}{3} \end{matrix}$

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Match 3 Round 5
Advanced Math:
Exponents and Logarithms

1.) ______ $\frac{x}{2}$ ______________

2.) ____________ $81, \frac{1}{9}$ ________

3.) _______ $128, \frac{1}{8}$ __________

1.) If $b^x = y$, what is $\log_{b^2}(y)$ in terms of x?

$$x = \frac{\log(y)}{\log(b)}. \quad \log_{b^2}(y) = \frac{\log(y)}{\log(b^2)} = \frac{\log(y)}{2\log(b)} = \frac{x}{2}$$

2.) Solve for all possible values of x:

$$(\log_3(x))^2 = \log_3(x^2) + 8$$

$$(\log_3(x))^2 - 2\log_3(x) - 8 = 0$$
$$(\log_3(x) - 4)(\log_3(x) + 2) = 0$$
$$\log_3(x) = 4\_or\_\log_3(x) = -2$$
$$x = 81, \frac{1}{9}$$

3.)_ If $z = \log_2(y)$, solve for all possible values of y:

$$\frac{64^{z-1}}{(0.125)^{z^2-4}} = 512^{2z+5}.$$

$$\frac{64^{z-1}}{(0.125)^{z^2-4}} = 512^{2z+5}$$

$$\frac{(2^6)^{z-1}}{(2^{-3})^{z^2-4}} = (2^9)^{2z+5}$$

$$\frac{2^{6z-6}}{2^{-3z^2+12}} = 2^{18z+45}$$

$$2^{(6z-6)-(-3z^2+12)=}2^{18z+45}$$

$$(6z-6)-(-3z^2+12) = 18z+45$$

$$3z^2+6z-18 = 18z+45$$

$$3z^2-12z-63 = 0$$

$$z^2-4z-21 = 0$$

$$(z-7)(z+3) = 0$$

$$z = 7, z = -3$$

$$z = \log_2(y)$$

$$y = 2^7, y = 2^{-3}$$

$$y = 128, \frac{1}{8}$$

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Match 3 Round 6
Discrete Math: Matrices

1.) ________17________________

2) ________2,3________________

3.) ____ $\begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ \frac{1}{4} & -\frac{1}{4} \end{bmatrix}$ ____

1. $C = \begin{bmatrix} 2 & 4 \\ -3 & 1 \\ 0 & 5 \end{bmatrix}$ and $D = \begin{bmatrix} 3 & -1 \\ -4 & 0 \\ 2 & 5 \end{bmatrix}$.

Give the sum of the six entries of $3C - 2D$.

$$3C = \begin{bmatrix} 6 & 12 \\ -9 & 3 \\ 0 & 15 \end{bmatrix}$$

$$2D = \begin{bmatrix} 6 & -2 \\ -8 & 0 \\ 4 & 10 \end{bmatrix}$$

$$3C-2D=\begin{bmatrix} 0 & 14 \\ -1 & 3 \\ -4 & 5 \end{bmatrix}$$

The sum of the entries is 17.

2) Find all values of k such that

$$\begin{vmatrix} k-1 & k+2 & 1 \\ 2 & k-4 & 1 \\ 3 & 2 & -1 \end{vmatrix} = 30$$

$$(k-1)(k-4)(-1)+(k+2)(1)(3)+1(2)(2)-$$

$$(3(k-4)(1)+2(1)(k-1)+-1(2)(k+2)=30$$

$$-k^2+8k+6-(3k-18)=30$$

$$-k^2+5k+24=30$$

$$-k^2+5k-6=0$$

$$k^2-5k+6=0$$

$$(k-2)(k-3)=0$$

$$k=2\_or\_k=3$$

3.) If A= $\begin{bmatrix} 4 & 3 \\ -2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$, find $(AB)^{-1}$

$$AB = \begin{bmatrix} 4(-1)+3(3) & 4(2)+3(-4) \\ (-2)(-1)+1(3) & (-2)(2)+1(-4) \end{bmatrix},$$

$$= \begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}$$ The determinant is 5(-8)-5(-4)=-20

$$(AB)^{-1} = \frac{1}{-20}\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ \frac{1}{4} & -\frac{1}{4} \end{bmatrix}$$

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FAIRFIELD COUNTY MATH LEAGUE 2017-18 Match 3 Team Round Solutions

Note: The inverse of a function or relation is not necessarily a function.

1.) ________249.F________$_{16}$

4.) _Domain: $x\geq 0$____Range: $y\geq 2$___

2.) _____ 54________minutes

5.)____________ $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ ________

3.) ___________7,13,17,29_______

6.) _______12______________

1.) In the base 16 (hexadecimal) system, A=10, B=11, C=12, D=13, E=14 and F=15. Express the base 10 number $\dfrac{(2*10^4)^7(8*10^7)(3*10^2)^9(5*10^3)}{(9*10^4)^4(4*10^4)^9}$ as a hexadecimal number.

$$\frac{(2*10^4)^7(8*10^7)(3*10^2)^9(5*10^3)}{(9*10^4)^4(4*10^4)^9}=$$

$$\frac{2^7*10^{28}*2^3*10^7*3^9*10^{18}*5*10^3}{(3^2)^4*10^{16}*(2^2)^9*10^{36}}=$$

$$\frac{2^{10}*10^{56}*3^9*5}{3^8*10^{16}*2^{18}*10^{36}}=\frac{3*5*10^4}{2^8}=\frac{3*5*5^4}{2^4}=\frac{9375}{16}=585\frac{15}{16}$$

585=2*256+4*16+9, so the number is 249.F

2.) Donald, Eric, and Ivanka work together to mow the lawn. The time it takes Donald to mow the lawn by himself is 3 times the amount of time it takes Eric to mow the lawn by himself. The time is takes Ivanka to mow the lawn by herself is 45 minutes more than the time it takes Eric to mow the lawn by himself. If all three work together, the lawn is done in 45 minutes. How long would it take to mow the lawn if only Eric and Ivanka work together?

Let x = amount of time it takes for Eric d to do the lawn. Then 3x = amount of time it takes Donald to mow the lawn and x+45=amount of time it takes Ivanka to mow the lawn, Then

$$\frac{45}{3x}+\frac{45}{x}+\frac{45}{x+45}=1$$

$$45(x+45)+45*3(x+45)+45(3x)=3x(x+45)$$

$$45x+2025+135x+6075+135x=3x^2+135x$$

$$3x^2-180x-8100=0$$

$$x^2-60x-2700=0$$

$$(x-90)(x+30)=0$$

$$x=90\_\min utes$$

$$3x=270\_\min utes$$

$$x+45=135\_\min utes$$

Let y=amount of time taken Eric and Ivanka work together.

$$\frac{y}{90}+\frac{y}{135}=1$$

$$3y+2y=270$$

$$5y=270, y=54\_\min utes$$

3.) The number of diagonals of a regular N-gon is the product of two distinct primes, p_1 and p_2, where $p_1 \neq p_2$. Find all such values of N where N≤30

Either N and $\frac{N-3}{2}$ are both prime or $\frac{N}{2}$ and N-3 are both prime. If N is prime, consider all the primes less than 30. If N=7, $\frac{N-3}{2}$=2, OK. Any other value of N must be of the form 4k+1 for an integer k in order for $\frac{N-3}{2}$ to be odd, so try 13, 17, and 29. If N=13, $\frac{N-3}{2}$=5, OK. If N=17,

$\dfrac{N-3}{2}$=7, OK. If N=29, $\dfrac{N-3}{2}$=13, OK. If $\dfrac{N-3}{2}$ is prime, $\dfrac{N-3}{2}$ must be of the form 4k+1 in order for N to be odd. Trying the same values for $\dfrac{N-3}{2}$ gives repeats of the values found earlier, so N = 7, 13, 17, 29

4.) If $f(x)=2^x$ and $g(x)=\log_4(x-2)$, what is the domain and range of the inverse of $y=f(g(x))$? If you use inequalities instead of interval notation, use x for the domain and y for the range.

$$f(g(x))=2^{\log_4(x-2)}=2^{\frac{\log_2(x-2)}{\log_2 4}}=(x-2)^{\frac{1}{2}}$$

To find the inverse, interchange x and y in $y=(x-2)^{\frac{1}{2}}$ and solve for y.

$x=(y-2)^{\frac{1}{2}}$ defined only for y≥2 and x≥0

$$x=(y-2)^{\frac{1}{2}}$$

$$x^2=y-2$$

$$y=x^2+2$$

for x≥0. y must be ≥2.

5.) If ABCBA = $\begin{bmatrix} -25 & 200 \\ 75 & 200 \end{bmatrix}$, A= $\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, and B = $\begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$, find matrix C.

$$A^{-1}=\frac{1}{10}\begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}=\begin{bmatrix} .4 & .2 \\ -.3 & .1 \end{bmatrix}$$

$$B^{-1}=\frac{1}{10}\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}=\begin{bmatrix} .1 & .3 \\ -.2 & .4 \end{bmatrix}$$

Multiply on the left side by $B^{-1}A^{-1}$ and on the right side by $A^{-1}B^{-1}$.

$$A^{-1}B^{-1}=\begin{bmatrix} 0 & .2 \\ -.05 & -.05 \end{bmatrix}$$

$$B^{-1}A^{-1}=\begin{bmatrix} -.05 & .05 \\ -.2 & 0 \end{bmatrix}$$

$$C=\begin{bmatrix} -.05 & .05 \\ -.2 & 0 \end{bmatrix}\begin{bmatrix} -25 & 200 \\ 75 & 200 \end{bmatrix}\begin{bmatrix} 0 & .2 \\ -.05 & -.05 \end{bmatrix}=$$

$$=\begin{bmatrix} 5 & 0 \\ 5 & -40 \end{bmatrix}\begin{bmatrix} 0 & .2 \\ -.05 & -.05 \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$$

6.) Solve for x: $\log_5(x-7)-\log_{25}(5x^2-95)=-1$

$$\log_5(x-7)-\log_{25}(5x^2-95)=-1$$

$$\log_5(x-7)-\frac{\log_5(5x^2-95)}{\log_5 25}=-1$$

$$\log_5(x-7)-\frac{\log_5(5x^2-95)}{2}=-1$$

$$2\log_5(x-7)-\log_5(5x^2-95)=-2$$

$$\log_5(x-7)^2-\log_5(5x^2-95)=-2$$

$$\log_5(\frac{x^2-14x+49}{x^2-95})=-2$$

$$\frac{x^2-14x+49}{5x^2-95}=\frac{1}{25}$$

$$\frac{x^2-14x+49}{x^2-19}=\frac{1}{5}$$

$$5(x^2-14x+49)=x^2-19$$

$$5x^2-70x+245=x^2-19$$

$$4x^2-70x+264=0$$

$$2x^2-35x+132=0$$

$$(x-12)(2x-11)=0$$

$$x=12, x=\frac{11}{2}$$

$$\frac{11}{2}\_is\_extraneous: x=12$$

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