FAIRFIELD COUNTY MATH LEAGUE 2017-2018

Match 1 Round 1
Arithmetic: Percents

1)____________3_____________

2.) ___________260___________

3.) _________8.3___________%_

1) (10N)% of 15 is $\frac{15}{2}$ less than 4N. Find N.

$$(\frac{10}{100}N)(15)=(4N)-\frac{15}{2}$$
$$(1.5)N=(4N)-\frac{15}{2}$$
$$-2.5N=-\frac{15}{2}$$
$$N=3$$

2) M% of 30 is P and P% of 400 is M+40. Find M+P.

$$(\frac{M}{100})(30)=P$$
$$(\frac{P}{100})(400)=M+40, so$$
$$4P=M+40, 0.3M=P$$
$$4(0.3M)=M+40$$
$$1.2M=M+40$$
$$0.2M=40$$
$$M=200$$
$$0.3M=0.3(200)=60$$
$$M+P=200+60=260$$

3.)_ 1% of a given population has a certain disease and 99% of the population does not have the disease. The test for the disease is 90% accurate: 90% of the time it gives the correct result and 10% of the time it gives the incorrect result. All members of the population are tested. To the nearest percent, what percent of the group that tests positive for the disease actually has the disease?

Say the population size is 1000. Then 990 do not have the disease and 10 do. 90% of the 10 (9) will test positive, as will 10% of the 990 (99). 108 people total will test positive, but only 9 out of the 108 have the disease. $\frac{9}{108}$ is about 8.3%.

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Match 1 Round 2
Algebra 1: Equations

1.) ______ $a = \frac{31}{7}$ ______________

2.) ______15________________

3.) ____ $b=2,-\frac{9}{2}$ ____________

1) Solve for a: $\frac{1}{5}a-4(3-a)=0.7(a+5)$

$\frac{1}{5}a-12+4a=0.7a+3.5$

$4.2a-12=0.7a+3.5$

$3.5a=15.5$

$\frac{7}{2}a=\frac{31}{2}$

$a=\frac{31}{7}$

2) x and y are natural numbers such that $1 \le x \le 100$ and $1 \le y \le 100$. How many distinct ordered pairs solve the equation $7x - 5y = 4$?

$7x-5y=4, so$

$y=\frac{7x-4}{5},$

$y=1.4x-0.8$

7x – 4 must be divisible by 5. x must come from the set {2,7,12,…97} but 7x-4 must be less than or equal to 500, so $7x \le 504$, and $x \le 72$, so the number of elements of {2,7,12,…67,72} is 15.

3) Solve for b: $b(b-0.5)(b+5.5)=(b-1.5)(b+3)(b+7)$.

$$b(b-0.5)(b+5.5)=(b-1.5)(b+3)(b+7)$$

$$b(b^2+5b-2.75)=(b-1.5)(b^2+10b+21)$$

$$b^3+5b^2-2.75b=b^3+8.5b^2+6b-31.5$$

$$4b^3+20b^2-11b=4b^3+34b^2+24b-126$$

$$20b^2-11b=34b^2+24b-126$$

$$14b^2+35b-126=0$$

$$2b^2+5b-18=0$$

$$(b-2)(2b+9)=0$$

$$b=2\_or\_b=\frac{-9}{2}$$

FAIRFIELD COUNTY MATH LEAGUE 2017-2018

Match 1 Round 3
Geometry: Triangles
And Quadrilaterals

1.)__________ 12__________cm

2.)__________ $5+5\sqrt{3}$___cm

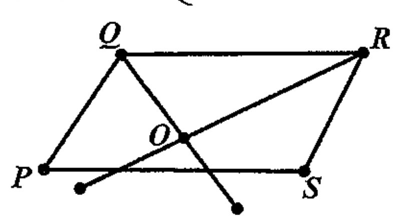
3.) _______ 56__________cm

1. In a right triangle, the hypotenuse is 7 cm longer than the shorter leg. The longer leg has length $\sqrt{119}$ cm. What is the length of the hypotenuse?

Let x=length of shorter leg, x+7 = length of hypotenuse.

$$(x+7)^2 - x^2 = (\sqrt{119})^2$$
$$x^2 + 14x + 49 - x^2 = 119$$
$$14x + 49 = 119$$
$$14x = 70$$
$$x = 5$$
$$x + 7 = 12$$

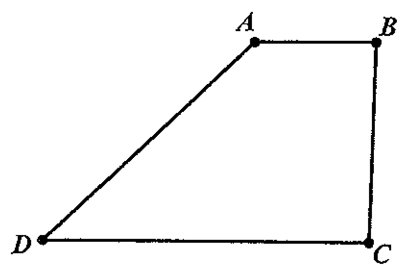
2. In parallelogram PQRS, angle P measures 60 degrees. The angle bisector of Q meets the angle bisector of R at point O. If QR=10 cm, find the sum of QO and RO.



The sum of angles PQR and QRS is 180 degrees, and since QO and RO are angle bisectors, the sum of angle OQR and ORQ is 90 degrees, so angle O is a right angle. Angle P measures 60 degrees, so does angle QRS. Angle

QRO is half of that, so it must be 30 degrees. ΔQOR is a 30-60-90 right triangle with hypotenuse 10 cm, so the short side must be 5 cm and the long side is $5\sqrt{3}$ cm.

3) In trapezoid ABCD as shown, $\overline{AB}$ is parallel to $\overline{CD}$ and the length of $\overline{CD}$ is 4 times the length of $\overline{AB}$. $\overline{AB}$ is perpendicular to $\overline{BC}$. The length of $\overline{BC}$ equals the length of $\overline{CD}$. If the area is 160 square cm, find the perimeter.



Let AB=x, and CD=BC=4x. The area is$(1/2)(x+4x)(4x)=10x^2 = 160$, so x=4. Then BC=4*4=16. Let a segment be drawn from A perpendicular to $\overline{CD}$ to meet $\overline{CD}$ at E. Then DE=3x, AE=4x, so AD=5x. The total perimeter is AB+BC+CD+DA=x+4x+4x+5x=14x=14*4=56.

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Match 1 Round 4
Algebra 2
Simultaneous
Equations

1.)___a=___$\frac{14}{5}$__ b= ___$\frac{3}{5}$__

2.)____k=____-6___

3.) __ $x=\frac{1}{3}, y=\frac{1}{4}, z=\frac{1}{5}$____

1.) Solve for a and b:

$$3a-4b=6$$
$$b=2a-5$$

$$3a-4(2a-5)=6$$
$$3a-8a+20=6$$
$$-5a=-14$$
$$a=\frac{14}{5}$$
$$b=2(\frac{14}{5})-5=\frac{3}{5}$$

2. For what values of k does the following system have infinitely many solutions (x,y)?

$$4x-ky=k+2$$
$$kx-9y=-k$$

$$4(-9)-k(-k)=0$$

We need $-36+k^2=0$

$$k=6\_or\_k=-6$$

If k=6, we have $\begin{matrix} 4x-6y=8 \\ 6x-9y=-6 \end{matrix}$ which has no solution.

If k=-6, we have $\begin{matrix} 4x+6y=-4 \\ -6x-9y=6 \end{matrix}$ which has infinitely many solutions.

3. Solve for x, y, and z:

$$\frac{1}{3x}+\frac{1}{4y}+\frac{2}{5z}=4$$

$$\frac{5}{6x}+\frac{1}{8y}+\frac{1}{z}=8$$

$$\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=4$$

Subtract equations 2 and 3

$$-\frac{1}{6x}+\frac{9}{8y}=4$$

Multiply equation 1 by 2.5 and subtract equation 3

$$\frac{-1}{6x}+\frac{13}{8y}=6$$

Subtract the two equations and solve for y:

$$\frac{-4}{8y}=-2, y=\frac{1}{4}$$

Substitute for y and solve for x:

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Match 1 Round 5
Trig
Right Triangles

1.)____ $\frac{28\sqrt{13}}{13}$ __________

2.)__ $50(\sqrt{3}-1)$_____ feet__

3.) _____$25\sqrt{241}$_________

1. In right triangle ABC, the right angle is at C. $\sin(\angle A)=\frac{\sqrt{13}}{7}$ and BC=4. Find AB.

$$\sin(\angle A)=\frac{\sqrt{13}}{7}=\frac{BC}{AB}=\frac{4}{AB}$$

$$(AB)(\sqrt{13})=28,$$

$$AB=\frac{28}{\sqrt{13}}=\frac{28\sqrt{13}}{13}$$

2. A kite is flying at an altitude lower than 100 feet. From the bottom of a 100 foot tall building, the angle of elevation to the kite is 45 degrees. From the top of the 100 foot tall building, the angle of depression to the kite is 60 degrees. What is the altitude of the kite in feet?

Let x= altitude of kite. y=horizontal distance from building to kite. We have

$$t\text{an}(60) = \frac{100-x}{y}$$

$$\frac{x}{\tan(45)} = \frac{100-x}{\tan(60)}$$

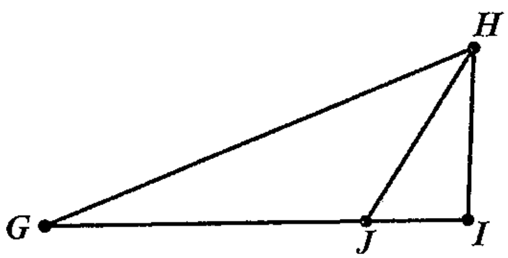
$$\frac{x}{1} = \frac{100-x}{\sqrt{3}}$$

$$x\sqrt{3} = 100 - x$$

$$x(\sqrt{3}+1) = 100$$

$$x = \frac{100}{\sqrt{3}+1} = \frac{100(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= 50(\sqrt{3}-1)$$



3. In the diagram above, $\overline{HI}$ is perpendicular to $\overline{GI}$.

HI = 100, GJ=250 and $\tan(\angle HJI)$ is three times the measure of $\tan(\angle HGI)$. Find GH.

$$\tan(\angle HJI) = \frac{100}{JI}$$

$$\tan(\angle HGI) = \frac{HI}{GI} = \frac{HI}{JI + 250}$$

$$3 * \frac{100}{JI + 250} = \frac{100}{JI}$$

$$\frac{100}{JI} = \frac{300}{JI + 250}$$

$$300(JI) = 100(JI + 250)$$

$$200(JI) = 25000$$

$$JI = 125.$$

$$GH = \sqrt{100^2 + 375^2} =$$

$$GH = \sqrt{4^2 * 25^2 + 15^2 * 25^2} =$$

$$GH = 25\sqrt{16 + 225} = 25\sqrt{241}$$

FAIRFIELD COUNTY MATH LEAGUE 2017-2018

Match 1 Round 6
Coordinate Geometry

1) $y=\frac{2}{3}x-\frac{22}{3}$

2) 9 , -15

3) ($-\frac{1}{12}$, $\frac{123}{16}$)

1.) Give the equation of the line parallel to 2x-3y=12 passing through the point (5,-4). Express your answer as y=mx+b for constants m and b

2x-3y=12 can be transformed to

$$2x-3y=12$$

$$-3y=-2x+12$$

$$y=\frac{2}{3}x-4$$

so the desired slope is $\frac{2}{3}$.

$$y+4=\frac{2}{3}(x-5)$$

$$y+4=\frac{2}{3}x-\frac{10}{3}$$

$$y=\frac{2}{3}x-\frac{22}{3}$$

2.) Find all values of x such that the length of the line segment connecting (-3, 4) and (x, -5) is 15.

$$\sqrt{(-3-x)^2+(4-(-5))^2}=15$$
$$(x+3)^2+9^2=15^2$$
$$(x+3)^2+81=225$$
$$(x+3)^2=144$$
$$x+3=12\_or\_x+3=-12$$
$$x=9\_or\_x=-15$$

3) An angle is formed by the two line segments from (2,3) to (5,7) and from (2,3) to (-4,11). Find the point where the perpendicular bisectors of these two segments intersect. Express your answer as an ordered pair (x,y).

The line from (2,3) to (5,7) has midpoint (3.5, 5) and slope $\frac{4}{3}$, the equation of the perpendicular bisector is

$$y=-\frac{3}{4}(x-3.5)+5\_or\_$$
$$y=-\frac{3}{4}x+\frac{21}{8}+5\_or\_$$
$$y=-\frac{3}{4}x+\frac{61}{8}$$

The line from (2,3) to (-4,11) has midpoint (-1,7) and slope $-\frac{4}{3}$, so the equation of the perpendicular bisector is

$$y=\frac{3}{4}(x+1)+7\_or\_$$
$$y=\frac{3}{4}x+\frac{3}{4}+7\_or\_$$
$$y=\frac{3}{4}x+\frac{31}{4}$$

Solve

$$\frac{3}{4}x+\frac{31}{4}=-\frac{3}{4}x+\frac{61}{8}$$

$$\frac{3}{2}x=\frac{61}{8}-\frac{62}{8}=-\frac{1}{8}$$

$$x=-\frac{1}{12}$$

$$y=\frac{3}{4}(-\frac{1}{12})+\frac{31}{4}=$$

$$-\frac{1}{16}+\frac{31}{4}=-\frac{1}{16}+\frac{124}{16}=\frac{123}{16}$$

FAIRFIELD COUNTY MATH LEAGUE 2017-18 Tm Round Match 1

1.)___ $\frac{11}{20}$ ________________

4.)___________29______________

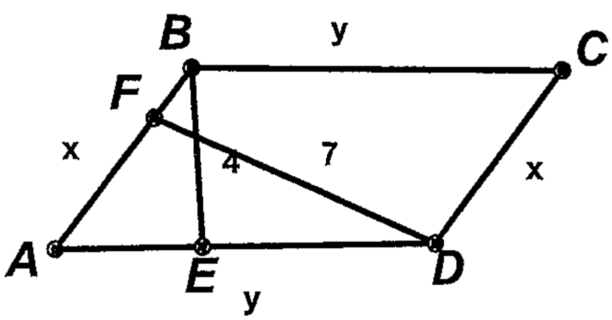
2.)____ $\frac{26}{3}$ _______

5.) _ $x \quad 5, y \quad 4; x \quad \frac{17}{5}, y \quad \frac{8}{5}$_

3.) Center:__(1,3)__Radius: __ $\sqrt{5}$ ___

6.) _______ $\frac{475}{34}\sqrt{5}$ ___________

The perimeter of a parallelogram is 40 cm and its altitudes measure 4 cm and 7 cm. Find the sine of any angle of the parallelogram.



$\sin(A) \quad \frac{7}{y}$_$by$_$\Delta\ ADF$

$\sin(A) \quad \frac{4}{x}$_$by$_$\Delta\ ABE2$

$2x \quad 2y \quad 40, x \quad y \quad 20, y \quad 20 \quad x$

$\sin(A)\quad \frac{7}{y}\_by\_\Delta\, ADF$

$\sin(A)\quad \frac{4}{x}\_by\_\Delta\, ABE$

$2x\quad 2y\quad 40, x\quad y\quad 20, y\quad 20\quad x$

$\frac{7}{20\quad x}\quad \frac{4}{x}, 80\quad 4x\quad 7x, x\quad \frac{80}{11}$

$\sin(A)\quad \frac{4}{\frac{80}{11}}\quad \frac{44}{80}\quad \frac{11}{20}$

2. A triangle has vertices at (0,0), (16,0), and (8,6). Find the length of the segment whose endpoints are its orthocenter and centroid.

Since the triangle is isosceles consisting of two sides of length 10, the median from (8,6) intersects the opposite side at (8,0). The centroid is two-thirds of the way from (8,6) to (8,0), so it is at (8,2). (To check this, you know that the centroid is on the line x=8, and look at another median, say the one from (0,0), which has goes through (0,0) and (12,3), so it has equation y=0.25x. When x=8, y=2.) . The altitude from (8,6) to the line through (0,0) and (16,0) is also x=8, since the line containing (0,0) and (16,0) is horizontal, therefore the altitude must be vertical. Consider the altitude from (0,0) to the line containing (16,0) and (8,6). The slope of this line segment is $\frac{3}{4}$ so the line from (0,0) to that side must be slope $\frac{4}{3}$. so the equation is $y\quad \frac{4}{3}x$. When x=8, y= $\frac{32}{3}$. The distance between the two points is just the difference in the y-coordinates since the x-coordinates are the same, so $\frac{32}{3}\quad 2\quad \frac{26}{3}$.

3. Find the center and radius of the circle passing through the points (2,1), (0,5), and (-1,2).

Several possible ways to solve, one way is simultaneous equations. If the center is (h,k) and the radius is r, then

$$(2 \quad h)^2 \quad (1 \quad k)^2 \quad r^2$$

$$(0 \quad h)^2 \quad (5 \quad k)^2 \quad r^2$$

$$(\; 1 \quad h)^2 \quad (2 \quad k)^2 \quad r^2$$

becomes

$$4 \quad 4h \quad h^2 \quad 1 \quad 2k \quad k^2 \quad r^2$$

$$h^2 \quad 25 \quad 10k \quad k^2 \quad r^2$$

$$1 \quad 2h \quad h^2 \quad 4 \quad 4k \quad k^2 \quad r^2$$

Subtract equations 1 and 3 to get $6h \quad 2k \quad 0, k \quad 3h$

$$(4 \quad 4h \quad h^2) \quad (1 \quad 6h \quad 9h^2) \quad r^2$$

$$(h)^2 \quad (25 \quad 30h \quad 9h^2) \quad r^2$$

$$(1 \quad 2h \quad h^2) \quad (4 \quad 12h \quad 9h^2) \quad r^2$$

$$From\_equation\_2$$

$$10h^2 \quad 30h \quad 25 \quad r^2$$

$$From\_equation\_3$$

$$10h^2 \quad 10h \quad 5 \quad r^2$$

$$Subtract$$

$$20h \quad 20 \quad 0, h \quad 1, k \quad 3$$

$$r^2 \quad 10(1)^2 \quad 10(1) \quad 5 \quad 5, r \quad \sqrt{5}$$

4. The senior class has 200 students. C% of the senior class takes Calculus, P% of the class takes Physics, (C-P+25) students take both, and 20% of the class takes neither. C, P, and C-P+25 must be whole numbers. What is the smallest possible value of C?

$$\frac{C}{100}200 \quad \frac{P}{100}200 \quad 0.2*200 \quad (C \quad P \quad 25) \quad 200$$

$$2C \quad 2P \quad 40 \quad C \quad P \quad 25 \quad 200$$

$$C \quad 3P \quad 185$$

185-C must be divisible by 3, so C belongs to {2,5,8,…98} since you can't exceed 100%. Since C-P+25 is a whole number, C must be greater than (P-25) The smallest possible value of C for which C is greater than or equal to P-25 and also satisfies C+3P=185 is C=29. (P=52, C-P+25=2)

5) Give the 2 ordered pair solutions for x and y: $\begin{matrix} 3y \quad 2x \quad 2 \\ x^2 \quad y^2 \quad 9 \end{matrix}$

$$x^2 \quad (\frac{2}{3}x \quad \frac{2}{3})^2 \quad 9$$

$$x^2 \quad \frac{4}{9}x^2 \quad \frac{8}{9}x \quad \frac{4}{9} \quad 9$$

$$\frac{5}{9}x^2 \quad \frac{8}{9}x \quad \frac{85}{9} \quad 0$$

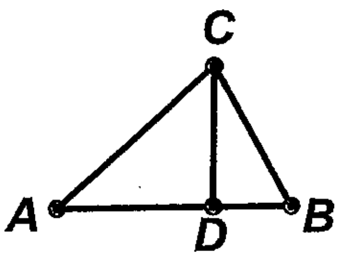
$$5x^2 \quad 8x \quad 85 \quad 0$$

$$(x \quad 5)(5x \quad 17) \quad 0$$

$$x \quad 5\_or\_x \quad \frac{17}{5}$$

$$If\_x \quad 5, y \quad 4$$

$$If\_x \quad \frac{17}{5}, y \quad \frac{34}{15} \quad \frac{2}{3} \quad \frac{24}{15} \quad \frac{8}{5}$$



6. In ΔABC, an altitude is drawn from C to $\overline{AB}$ at D. , $\cos(\quad CBA)\ \frac{3}{5}$, $\tan(\quad CAB)\ \frac{1}{4}$ and AC=10. What is the area of ΔABC?

If CD=x, then AD=4x, so we have $x^2 + (4x)^2 = 100$

$x^2 = \frac{100}{17}$, $x = \frac{10\sqrt{17}}{17}$.

Since $\cos(\quad CBA)\ \frac{3}{5}$, BD= $\frac{3x}{4}$, AD+BD=4x+ $\frac{3x}{4} = \frac{19x}{4}$

Area is 0.5(AD+DB)*CD=

$$\frac{1}{2}(\frac{19}{4} * \frac{10\sqrt{17}}{17})(\frac{10\sqrt{17}}{17})$$

$$\frac{1}{2}(\frac{190\sqrt{17}}{68})(\frac{10\sqrt{17}}{17})$$

$$\frac{1}{2}(\frac{1900}{68})\quad \frac{4*475}{4*34}\quad \frac{475}{34}$$