FAIRFIELD COUNTY MATH LEAGUE (FCML) 2016-2017

Match 3 Round 1 Arithmetic: Scientific Notation and Bases

1.) ____________36________

2.) __________12123_5________

3.) _______2,4,9,11__________

1.) When $(((4 \times 10^4)^2)^2)^2$ is expressed in scientific notation $M \times 10^n$ where $1 \leq M < 10$, what is the exponent n?

$$(((4 * 10^4)^2)^2)^2 =$$
$$((16 * 10^8)^2)^2$$
$$= (256 * 10^{16})^2$$
$$= 65536 * 10^{32} =$$
$$6.5536 * 10^{36}$$

2.) Simplify $(1234_9 - 2341_8) + 3421_7$.
Give your answer as a base five expression
1*729+2*81+3*9+4 = 922
2*512+3*64+4*8+1 = 1249
3*343+4*49+2*7 + 1 = 1240
922-1249+1240 = 913
913-1*625=288
288-2*125=38
38-1*25=13
13-2*5=3
so 12123_5

3.)_ b is a whole number base between 2 and 12 inclusive. For which values of b is the expression 111_b divisible by 7?

You can make a table:

b	b^2+b+1	Divisible by 7 ?
2	7	Yes
3	13	No
4	21	Yes
5	31	No
6	43	No
7	57	No
8	73	No
9	91	Yes
10	111	No
11	133	Yes
12	157	No

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Match 3 Round 2
Algebra: Word Problems

1.) _________250_________________

2.) _____________ $\frac{5}{11}$ __________meters

3.) __x= __160_____ml, y= ___25__%___

1.) The senior class sold tickets for the school play. Tickets were \$8 for students were \$12 for adults. 400 people attended the play and the senior class made \$3800. How many students came to the play?

Let x = # students , 400-x = # of adults

8x + 12(400-x)=3800, -4x+4800=3800, -4x=-1000, x=250

2.) Older sister Judy and little brother Johnny are having a race to a pole that is 25 meters from Judy's starting point, and then back to Judy's starting point. Judy runs at 12 m/s, but since Johnny only runs at 10 m/s, she gives him a 5 meter headstart before they start running. Judy runs toward the pole and meets Johnny coming back from the pole. How far from the pole are they when this happens?

At time=2 seconds, Johnny is at the pole and turning around, and Judy is 1 meter from the pole heading towards the pole. If they must meet at the same place at the same time, if the time it takes Judy to meet Johnny, she must have run 12t meters, and Johnny runs 1 – 12t meters, but this is also equal to 10t meters, so 1-12t=10t and $t=\frac{1}{22}$ seconds. The distance from the pole is $10t=10(\frac{1}{22})=\frac{5}{11}$ meters.

3) If you add 40 ml of a solution of 80% acid to x ml of solution that has y % of acid, the resulting solution has a concentration 36% acid. If you had added 40 ml of solution of 20% acid to the solution of x ml at y% acid , the concentration would have been 24%. Find x and y. Do not write an additional % sign when giving your answer for y.

Amount of solution in each case is x+40

Acid equation in the first case is $0.8(40)+\frac{y}{100}x=0.36(x+40)$

Acid equation in the first second case is $0.2(40)+\frac{y}{100}x=0.24(x+40)$, so $\frac{y}{100}x$ in each case is $0.36(x+40)-32$ and $0.24(x+40)-8$, so they equal each other.

$$0.36(x+40)-32=0.24(x+40)-8$$

$$0.36x+14.4-32=0.24x+9.6-8$$

$$0.36x-17.6=0.24x+1.6$$

$$0.12x=19.2, x=160$$

$$\frac{y}{100}(160)=0.36(160+40)-32$$

$$1.6y=0.36(200)-32$$

$$1.6y=72-32=40$$

$$y=25$$

3.)

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Match 3 Round 3
Geometry: Polygons

1.) ________45______________

2.) ________27____________

3.) _______11_______________

1.)The measure of one interior angle of a regular polygon is 172 degrees. How many sides does the polygon have?

$$172=\frac{180(n-2)}{n},172n=180n-360,8n=360,n=45$$

2.)The number of diagonals of a convex hexagon is 9, and 9 is a perfect square. Find one other convex N-gon with N<30 such that the number of diagonals is a perfect square.

By trial and error and recognizing that you will need some factors of N and N-3 that can multiply to twice a perfect square, N=27, since

$$\frac{27(27-3)}{2}=324=18^2 \quad \frac{27(27-3)}{2}=324=18^2$$

3.) M angles of an N-gon measure 120 degrees and the rest measure 144 degrees. If M and N are both greater than 0 and M<N, find the value of M+N.

M*120 + (N-M)*144=180(N-2)
M*120 + 144N – 144M = 180N -360
-24M -36N = -360
24M + 36N = 360
12(2M+3N) = 360
2M+3N = 30

M and N must be natural numbers, so the only possibilities are when $\frac{30-3N}{2}$ is a whole number and M<N. N must be even for 30-3N to be divisible by 2. If N=2, you don't have a polygon. If N=4 or 6, you don't have M<N. If N=8, then M=3, and N-M=5. If N=10, then M is not greater than 0. M+N=11.

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Match 3 Round 4 Algebra 2: Functions and Inverses

1.) __$f^{-1}(x)=$__ $\frac{-2x}{3x+2}$ ______

Note: The inverse of a function is not necessarily itself a function.

2). $5 < g(x) < \infty$ _or_ $-\infty < g(x) \le \frac{-1}{2}$

3.) ________ $x \ge \frac{3}{4}$ ____________

1.) $f(x)=\frac{-2x}{3x+2}$. Find $f^{-1}(x)$. Express your answer as $f^{-1}(x)=$....

Let $y=\frac{-2x}{3x+2}$

Interchange x and y and solve for y.

$x=\frac{-2y}{3y+2}$

$3xy+2x=-2y$

$3xy+2y=-2x$

$y(3x+2)=-2x$

$y=\frac{-2x}{3x+2}$

$f^{-1}(x)=\frac{-2x}{3x+2}$

The function is its own inverse.

2.) If $g(x)=\frac{5x^2+2}{x^2-4}$, give the range of g(x). Either use interval notation or use g(x) in your inequality.

g(x) has asymptotes at x=2 and x=-2. If x is close to negative 2 on the left or close to positive on the right, g(x) is a large positive number, so g(x) increases without bound as x gets close to 2 from the right or -2 from the left. As x approaches -2 from the right or 2 from the left, g(x) decreases without bound. As x gets large negative or large positive, g(x) approaches 5. For -2<x<2, g(x) is greatest when x=0. When x=0, g(x)= $\frac{-1}{2}$. The range is $5<g(x)<\infty\_or\_-\infty<g(x)\leq\frac{-1}{2}$

3.) p(x)=x^2 +5x + 7. q(x)=3x+11. If h(x) =p(q(x)), give the domain of $h^{-1}(x)$. Either use interval notation or use the variable x in your inequality.

$$p(x)=x^2+5x+7, q(x)=3x+11$$

$$p(q(x))=(3x+11)^2+5(3x+11)+7=$$

$$9x^2+66x+121+15x+55+7=$$

$$9x^2+81x+183$$

$$h(x)=9x^2+81x+183$$

Find the vertex by completing the square on h(x).

$$h(x)=9x^2+81x+183$$

$$h(x)=9(x^2+9x)+183$$

$$h(x)=9(x^2+9x+20.25)+183-182.25$$

$$h(x)=9(x+4.5)^2+0.75$$

The vertex of h(x) is at (-4.5, 0.75) , so the vertex of $h^{-1}(x)$ is (0.75,-4.5) and it opens to the right so x≥0.75

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Match 3 Round 5 Advanced Math: Exponents and Logarithms

1.) ______ 1.486______________

2.) ____________ $\frac{4}{3}$ ____________

3.) _______ $27, \frac{\sqrt[4]{27}}{3}$ ___________

1.) If $\log_{10} 3 = 0.477$, and $\log_{10} 5 = 0.699$, and $\log_{10} 7 = 0.845$, find $\log_{10}(\frac{1500}{49})$

$$\log_{10}(\frac{1500}{49}) = \log_{10}(\frac{3*5*100}{7*7})$$
$$= \log(5) + \log(3) + \log(100) - \log(7) - \log(7)$$
$$= 0.699 + 0.477 + 2 - 0.845 - 0.845 = 1.486$$

2.) Simplify as much as possible:

$$(\log_7 3)(\log_{27} 4)(\log_5 49)(\log_4 25)$$

$$= \frac{\log 3}{\log 7} * \frac{\log 4}{\log 27} * \frac{\log 49}{\log 5} * \frac{\log 25}{\log 4}$$
$$= \frac{\log 3}{\log 7} * \frac{\log 4}{3\log 3} * \frac{2\log 7}{\log 5} * \frac{2\log 5}{\log 4} = \frac{4}{3}$$

3.)_ If $z=\log_3(y)$, solve for all possible values of y:

$$\frac{25^{2z^2+2}}{(0.2)^{z-1}} = 125^{4z+2}.$$

Express any radical answers in simplest radical form.

$$25^{2z^2+2} \div (0.2)^{z-1} = 125^{4z+2}$$

$$(5^2)^{2z^2+2} \div (5^{-1})^{z-1} = (5^3)^{4z+2}$$

$$4z^2 + 4 - (-1)(z-1) = 3(4z+2)$$

$$4z^2 + 4 + z - 1 = 12z + 6$$

$$4z^2 - 11z - 3 = 0$$

$$(4z+1)(z-3) = 0$$

$$z = \frac{-1}{4}, z = 3$$

$$z = \log_3(y)$$

$$3 = \log_3(y), so\_y = 27$$

$$\frac{-1}{4} = \log_3(y), so\_y = \frac{1}{\sqrt[4]{3}} = \frac{\sqrt[4]{27}}{3}$$

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Match 3 Round 6
Discrete Math: Matrices

1.) ________ -4 ________________

2.) ______ $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ ________

3.) ____________ ~~2~~ 6 ____________

1.) Find all values of k such that

$$\begin{bmatrix} k & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} k & -1 \\ 3 & -k \end{bmatrix}=\begin{bmatrix} 25 & 2k+24 \\ k-1 & 2 \end{bmatrix}$$

$k^2+9=25,\_so\_k=4\_or\_k=-4$

$but\_-k-3k=2k+24\_and\_2k+3=k-1\_and\_-2-k=2\_so$

$k=-4$

2.) If A= $\begin{bmatrix} 5 & -4 \\ -6 & 5 \end{bmatrix}$ and ABA = $\begin{bmatrix} -6 & 7 \\ 8 & -9 \end{bmatrix}$, find B

Multiply ABA by A^{-1} on the right and on the left.

$$A^{-1}=\begin{bmatrix} 5 & 4 \\ 6 & 5 \end{bmatrix},$$

$$A^{-1}ABAA^{-1}=\begin{bmatrix} 5 & 4 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} -6 & 7 \\ 8 & -9 \end{bmatrix}\begin{bmatrix} 5 & 4 \\ 6 & 5 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & -1 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 5 & 4 \\ 6 & 5 \end{bmatrix}=\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

3.) Give the sum of the nine entries of the inverse of

$$\begin{bmatrix} 0 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Find the determinant: $0\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} =$

$$0-2(36-42)+3(32-35)=12-9=3$$

The transpose is

$$\begin{bmatrix} 0 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$ so the inverse is

$$\frac{1}{3}\begin{bmatrix} \begin{vmatrix} 5 & 8 \\ 6 & 9 \end{vmatrix} & -\begin{vmatrix} 2 & 8 \\ 3 & 9 \end{vmatrix} & \begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} \\ -\begin{vmatrix} 4 & 7 \\ 6 & 9 \end{vmatrix} & \begin{vmatrix} 0 & 7 \\ 3 & 9 \end{vmatrix} & -\begin{vmatrix} 0 & 4 \\ 3 & 6 \end{vmatrix} \\ \begin{vmatrix} 4 & 7 \\ 5 & 8 \end{vmatrix} & -\begin{vmatrix} 0 & 7 \\ 2 & 8 \end{vmatrix} & \begin{vmatrix} 0 & 4 \\ 2 & 5 \end{vmatrix} \end{bmatrix}$$

$$\frac{1}{3}\begin{bmatrix} -3 & 6 & -3 \\ 6 & -21 & 12 \\ -3 & 14 & -8 \end{bmatrix}$$

so the sum is $\frac{1}{3}(-3+6-3+6-21+18-3+14-8)=\frac{1}{3}(6)=2$

FAIRFIELD COUNTY MATH LEAGUE 2016-17 Match 3 Team Round Solutions

Note: The inverse of a function or relation is not necessarily a function.

1.) _______10.B_______12 4.) ________-4_______________

2.) _____ 24________minutes 5.)_______________4__________

3.) ___________22_______ 6.) _______(8,10)_____________

1.) In base 12, A=10 and B=11. Simplify the following decimal expression and express your answer in base 12. $\frac{(3100x10^8)(2000x10^{-12})}{(2400x10^6)(0.2x10^{-7})}$

$$\frac{(3100x10^8)(2000x10^{-12})}{(2400x10^6)(0.2x10^{-7})}$$

$$=\frac{3.1*10^3*10^8*2*10^3*10^{-12}}{2.4*10^3*2*10^{-1}*10^{-1}}=\frac{3.1x10^2}{2.4x10^1}=$$

$$\frac{310}{24}=12\frac{11}{12}=1*12^1+0*12^0+11*12^{-1}=10.B_{12}$$

2.) .NEW: Moe deLaune can mow the lawn in 40 minutes. If his friend Larry helps him, it takes 5 minutes less time than if his friend Curly helps him. The time it takes Larry to mow the lawn by himself is 16 minutes less than the time it takes Curly to mow than the lawn by herself. How long would it takes Larry to mow the law by himself?

Moe does $\frac{1}{40}$ lawn in one minute. Let x = amount of time it takes for Moe to do the lawn with Larry helping him. If Larry mows the lawn by himself in L minutes, then together $\frac{x}{40}+\frac{x}{L}=1$. If Curly mows the lawn by himself in C minutes, than $\frac{x+5}{40}+\frac{x+5}{C}=1$. But L=C-16, so $\frac{x+5}{40}+\frac{x+5}{L+16}=1$. From the first equation, $x=\frac{40L}{40+L}$, so

$$\frac{\frac{40L}{40+L}+5}{40}+\frac{\frac{40L}{40+L}+5}{L+16}=1$$

$$\frac{\frac{45L+200}{40+L}}{40}+\frac{\frac{45L+200}{40+L}}{L+16}=1,$$

$$\frac{45L+200}{40(40+L)}+\frac{45L+200}{(40+L)(L+16)}=1$$

$$\frac{(45L+200)(L+16)+(45L+200)40}{40(40+L)(L+16)}=1$$

$$\frac{45L^2+920L+3200+1800L+8000}{40(40+L)(L+16)}=1$$

$$\frac{45L^2+920L+3200+1800L+8000}{40(40+L)(L+16)}=1$$

$$45L^2+920L+3200+1800L+8000=40L^2+2240L+25600$$

$$45L^2+2720L+11200=40L^2+2240L+25600$$

$$5L^2+480L-14400=0$$

$$L^2+96L-2880=0$$

$$(L-24)(L+120)=0$$

$$L=24$$

3.) For how many values of N does a regular N-gon have an interior angle that is a whole number of degrees?

If the interior angle is a whole number of degrees, so is the exterior angle. Then it comes down to most of the factors of 360, beginning with 120 for the exterior angle of an equilateral triangle and then working down, so we have
120 (n=3), 90 (n=4), 72 (n=5), 60 (n=6), 45 (n=8), 40 (n=9), 36 (n=10), 30 (n=12), 24 (n=15), 20 (n=18) so that makes 10 factors of 360 greater than the square root of 360, and then take all of the values for which these numbers multiply to 360 (10 more), plus an exterior angle measure of 2 (n=180) and an exterior angle of 1 (n=360), which gives 22.

4.) Find all values of x such that the matrix $\begin{bmatrix} 2^{2x} & 4^{x+2} \\ 8^{-\frac{1}{3}x+2} & 0.5^{3x-2} \end{bmatrix}$ does not have an inverse.

We need the determinant equal to zero, so

$$(2^{2x})(0.5^{3x-2})-(8^{-\frac{1}{3}x+2})(4^{x+2})=$$

$$(2^{2x})(2^{-1})^{3x-2}-(2^{3})^{-\frac{1}{3}x+2}(2^{2})^{x+2}=$$

$$2^{2x-3x+2}-(2^{-x+6})(2^{2x+4})$$

\

$$2^{-x+2}-2^{x+10}=\frac{4}{2^{x}}-1024*2^{x}=0$$

$$4=1024*2^{x}2^{x}$$

$$\frac{1}{256}=2^{2x},2^{-8}=2^{2x},2x=-8,x=-4$$

5.) Solve for x: $\log_9(4x^2+17)-\log_3(8x-5)=-1$

$$\log_9(4x^2+17)-\log_3(8x-5)=-1$$

$$\frac{\log(4x^2+17)}{\log 9}-\frac{\log(8x-5)}{\log 3}=-1$$

$$\frac{\log(4x^2+17)}{2\log 3}-\frac{\log(8x-5)}{\log 3}=-1$$

$$\frac{\log(4x^2+17)}{2\log 3}-\frac{2\log(8x-5)}{2\log 3}=-1$$

$$\log(\frac{4x^2+17}{(8x-5)^2})=-2\log(3)=$$

$$(\frac{4x^2+17}{64x^2-80x+25})=\frac{1}{9}$$

$$64x^2-80x+25=9(4x^2+17)$$

$$64x^2-80x+25=36x^2+153$$

$$28x^2-80x-128=0$$

$$7x^2-20x-32=0$$

$$(x-4)(7x+8)=0$$

$$x=4\_or\_x=\frac{-8}{7}$$

but $\frac{-8}{7}$ is extraneous, so x=4

6.) $f(2x)=2\sqrt{x-1}+4$ and $g(\frac{x}{3})=\frac{1}{4}(x+2)$. Find all points (x,y) where $f^{-1}(x)$ and $g^{-1}(x)$ intersect.

$$f(2x)=2\sqrt{x-1}+4$$

$$f(x)=2\sqrt{(\frac{x}{2})-1}+4=2\sqrt{\frac{x-2}{2}}+4$$

$$g(\frac{x}{3})=\frac{1}{4}(x+2)$$

$$g(x)=\frac{1}{4}(3x+2)=\frac{3}{4}x+\frac{1}{2}$$

$$f(x)=g(x)\_when\_2\sqrt{\frac{x-2}{2}}+4=\frac{3}{4}x+\frac{1}{2}$$

$$2\sqrt{\frac{x-2}{2}}=\frac{3}{4}x-\frac{7}{2},\sqrt{\frac{x-2}{2}}=\frac{3}{8}x-\frac{7}{4},$$

$$(\sqrt{\frac{x-2}{2}})^2=(\frac{3}{8}x-\frac{7}{4})^2$$

$$\frac{x-2}{2}=\frac{9}{64}x^2-\frac{21}{16}x+\frac{49}{16}$$

$$\frac{9}{64}x^2-\frac{29}{16}x+\frac{65}{16}=0$$

$$9x^2-116x+260=0$$

$$(x-10)(9x-26)=0$$

$$x=10\_or\_x=\frac{26}{9}$$

$$If\_x=10,f(x)=2\sqrt{\frac{10-2}{2}}+4=8$$

$$If\_x=\frac{26}{9},$$

this solution is extraneous, since just before the step where we squared both sides, $x = \frac{26}{9}$ gives a positive number on one side and a negative number on the other side. When you substitute $x = \frac{26}{9}$ for f(x), you get $\frac{16}{3}$, and when you substitute $x = \frac{26}{9}$ for g(x) you get $\frac{8}{3}$. So the only solution for the intersection of f(x) and g(x) is (10,8), so the solution for the intersection of $f^{-1}(x)$ and $g^{-1}(x)$ is (8,10).