FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 3 Round 1
Arithmetic: Scientific Notation and Bases

1.) ________ 2.5×10^{-5} ________

2.) ____________ 1285 _________

3.) ________ 5, 8, 10, 11, 12 _____

1.) Simplify and express your answer in scientific notation:

$$\frac{(4x10^{5})(3x10^{-7})}{(8x10^{-6})(6x10^{8})}$$

$$\frac{12x10^{-2}}{48x10^{2}} = 0.25x10^{-4} = 2.5x10^{-5}$$

2.) Add the numbers 1232_8 , 1232_4 , and 1232_5. Give your answer in base 9.

1232_8 = 512+128+24+2=666 1232_4 = 64+32+12+1= 110 1232_5 = 125+50+15+2= 192 666+110+192=968. 968 = 729+2*81+8*9+5, so it's
1285_9

3.)_b is a whole number base between 2 and 12 inclusive. For which values of b is the expression $\frac{1001_b}{11_b}$ NOT a prime number?

You can make a table:

2	9	3	3	prime
3	28	4	7	prime
4	65	5	13	prime
5	126	6	21	not prime
6	217	7	31	prime
7	344	8	43	prime
8	513	9	57	not prime
9	730	10	73	prime

10	1001	11	91	not prime
11	1332	12	111	not prime
12	1729	13	133	not prime

Or if you've taken Algebra 2, you can recognize that you have the expression $\frac{b^3+1}{b+1}=b^2-b+1$ and substitute the values 2 through 12 into b^2-b+1.

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Match 3 Round 2
Algebra: Word Problems

1.) ________20________________

2.) ________________1858________________________

3.) __ morning: _50___ mph afternoon: __30_____ mph

1.) Susan has a collection of 50 coins, all dimes and quarters. The total value of the coins is \$9.50. How many dimes does Susan have?

Let x = # dimes , 50-x = # of quarters.
10x + 25(50-x)=950. 10x+1250-25x=950. -15x=-300 . X=20.

2) Theodore and Franklin were two presidents. When Theodore was inaugurated as president in 1901, his age in years was 5 more than twice Franklin's age. When Theodore died in 1919, his age was 50 less than three times Franklin's age. In what year was Theodore born?

Let t=Theodore's age in 1901, f= Franklin's age in 1901.
t=2f+5. 18 years later, we have t+18 = 3(f+18)-50.
So 2f+5+18 = 3f+54-50
2f+23=3f+4
f=19, and t=43. Subtract 43 from 1901 to get 1858.

3.) Bill drives 10 miles from Easton to Westport in the morning. He encounters more traffic coming home so his average speed for the afternoon trip is 20 mph less than his average speed for the morning. If the total commuting time is 32 minutes, what are Bill's average speeds for each part of the trip? Give your answers in miles per hour.

Let r+20 = Bill's average speed for the morning, r= Bill's average speed for the afternoon. The total time is 32 minutes or $\frac{8}{15}$ hour.

Time = Distance / Rate, so

$$\frac{10}{r}+\frac{10}{r+20}=\frac{8}{15}$$

Multiply each side by r(r+20)15 to get

$$150(r+20)+150r=8r(r+20)$$

$$150r+3000+150r=8r^2+160r$$

$$8r^2-140r-3000=0$$

$$2r^2-35r-750=0$$

$$(r-30)(2r+25)=0$$

$$r=30, r+20=50$$

The -25/2 answer is extraneous.

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Match 3 Round 3
Geometry: Polygons

1.) ________30____________

2.) ________ 9__________

3.) ____7, 8, 9, 10___________

1.) The number of diagonals of a regular polygon is 405. How many sides does the polygon have?

$$405=\frac{n(n-3)}{2}, n(n-3)=810, n^2-3n-810=0$$

$$(n-30)(n+27)=0, \therefore n=30$$

2.) Three of the interior angles of a convex polygon add to 300 degrees. All of the other interior angles are congruent and each of these angles is two-thirds of the sum of the exterior angles associated with the three angles that add to 300 degrees. How many sides does the polygon have?

The combined interior and exterior angles of the three exterior angles associated with the angles that add to 300 degrees is 3*180=540 degrees, so the three exterior angles add to 540-300=240 degrees. The congruent interior angles are each two-thirds of 240 = 160 degrees. We have 180(n-2)=300+(n-3)*160, so 180n-360=300+160n-480
20n-360=300-480, 20n-360=-180, 20n=180, so n=9.

3.) The interior angle of a regular M-gon and the interior angle of a regular N-gon sum to 300 degrees. If M<N, there are four combinations of M and N for which this is true. Give the four possible values of M.

$$\frac{180(m-2)}{m}+\frac{180(n-2)}{n}=300$$

$Multiply\_by\_mn$

$$180(m-2)n+180(n-2)m=300mn$$

$$60mn-360n-360m=0$$

$$mn-6n-6m=0$$

$$Now\_n=\frac{6m}{m-6}$$

When n=12 and m=12, both polygons have interior angles 150 degrees, so we need whole number values of m<12 such that $\frac{6m}{m-6}$ is a whole number.

n<0 for m<6 and undefined for m=6.
If m=7, n-42. If m=8, n=24. If m=9, n=18. If m=10, n=15. If m=11, n is a not a whole number.

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Match 3 Round 4
Algebra 2: Functions and Inverses

1.) _____4______________

Note: The inverse of a function is not necessarily itself a function.

2). .) ___$x \geq \frac{27}{4}$_________________

3.) _Domain:_$1 \leq x < 10$ or $x > 10$_ Range:_$y \leq -2$ or $y > 0$_____

3

1.) If f(x) = 5x+4 and g(x) = 2x-6, find $g^{-1}(f^{-1}(14))$.

$f^{-1}(x) = \frac{x-4}{5}$. $g^{-1}(x) = \frac{x+6}{2}$

$f^{-1}(14) = \frac{14-4}{5} = 2$ $g^{-1}(2) = \frac{2+6}{2} = 4$

2.)_$g(x)=x^2 - 3x + 9$. Give the domain of the relation $g^{-1}(x)$. Either use interval notation or use the variable x in your inequality.

g(x)=. Find the vertex by completing the square g(x) =

$x^2 - 3x + 2.25 + 9 - 2.25 =$

$(x-1.5)^2 + 6.75$

The vertex of g(x) is at $(\frac{3}{2}, \frac{27}{4})$, so the vertex of $g^{-1}(x) = (\frac{27}{4}, \frac{3}{2})$ and it opens to the right, so the domain is $x \geq \frac{27}{4}$

3.) What is the domain and range of $y = \frac{6}{\sqrt{x-1} - 3}$? Either use interval notation or use x for your inequality describing the domain and use y for your inequality describing the range.

x must be ≥ 1 in order to be able to take the square root, but it can't be equal to 10, since the denominator would then be zero. So $1 \leq x < 10$ or $x > 10$. When $x=1$, $y=-2$. As x increases from 1, y decreases without limit until x reaches 10; then as you approach 10 from the right, y increases with limit, and as x increases from 10, y approaches 0, so $y \leq -2$ or $y > 0$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 3 Round 5
Advanced Math:
Exponents and Logarithms

1.) 2.498

2.) $\frac{1}{25}, \frac{1}{125}$

3.) 5

1.) If $\log_{10} 3 = 0.477$, and $\log_{10} 5 = 0.699$, and $\log_{10} 7 = .845$, find $\log_{10}(315)$

$$\log_{10}(315) = \log_{10}(3*3*5*7)$$
$$= \log(3) + \log(3) + \log(5) + \log(7)$$
$$= 0.477 + 0.477 + 0.699 + 0.845 = 2.498$$

2.) Find all possible values of z if $\log_5(z) = y$ and $(4^{y^2+2y})(8^{y-3}) = (0.5)^{(3y+21)}$

$$(2^{2(y^2+2y)})(2^{3(y-3)}) = (2)^{-(3y+21)}$$
$$2y^2 + 4y + 3y - 9 = -3y - 21$$
$$2y^2 + 10y + 12 = 0$$
$$y^2 + 5y + 6 = 0$$
$$(y+2)(y+3) = 0$$
$$y = -2\_or\_y = -3$$
$$z = \frac{1}{25}, \frac{1}{125}$$

3.) Find all values of x such that

$$x25^{\log_5(x+3)} - (64^{\log_4(x+2)}) = -23$$

$$x25^{\log_5(x+3)} - (64^{\log_4(x+2)}) = -23$$

$$x25^{\log_5(x+3)} = x5^{2\log_5(x+3)} = x(5^{\log_5(x+3)})^2 =$$

$$x(x+3)^2 = x^3 + 6x^2 + 9x$$

$$64^{\log_4(x+2)} = 4^{(3\log_4(x+2))} = (4^{(\log_4(x-3))})^3 = (x+2)^3 =$$

$$x^3 + 6x^2 + 12x + 8$$

$$x^3 + 6x^2 + 9x - (x^3 + 6x^2 + 12x + 8)$$

$$= -3x - 8$$

$$Solve\_ -3x - 8 = -23$$

$$-3x = -15\_ x = 5$$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 3 Round 6
Discrete Math: Matrices

1.) -39

2.) $\begin{bmatrix} 17 & 4 \\ -6 & -35 \end{bmatrix}$

3.) 2

1.) Give the sum of the four entries of the matrix product:

$$\begin{bmatrix} 2 & -2 & 3 \\ -4 & 5 & 1 \end{bmatrix} \begin{bmatrix} 6 & 4 \\ -3 & 2 \\ 1 & -5 \end{bmatrix}$$

$a_{11}=12+6+3=21$

$a_{12}= 8 -4 - 15 = -11$

$a_{21}=(-24)+(-15)+1 = -38$

$a_{22}=(-16)+10+(-5)=-11$

Sum is 21-11-38-11 = -39

2.) If $A=\begin{bmatrix} 5 & 4 \\ 6 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, find $AB^{-1} + A^{-1}B$

Both matrices have determinant 1.

$$A^{-1} = \begin{bmatrix} 5 & -4 \\ -6 & 5 \end{bmatrix}, B^{-1} = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

$$AB^{-1} = \begin{bmatrix} 5 & 4 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 6 & -13 \\ 7 & -15 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} 5 & -4 \\ -6 & 5 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 11 & 17 \\ -13 & -20 \end{bmatrix}$$

$$Sum\_is\_\begin{bmatrix} 17 & 4 \\ -6 & -35 \end{bmatrix}$$

3 .) Find all values of k such that the determinant of

$$\begin{bmatrix} k & k-4 & k+3 \\ k-1 & k & k+1 \\ 2 & -1 & 1 \end{bmatrix}$$

is -25.

We need k*k*1+(k-4)(k+1)*2 + (k+3)(k-1)(-1) –
[2k(k+3) + -1(k+1)k + 1(k-1)(k-4) = -25

$k^2 + 2(k-4)(k+1) -(k+3)(k-1) - [2k^2 + 6k + (-k^2 - k) + k^2 -5k + 4] = -25$
$k^2 + 2k^2 - 6k - 8 - k^2 - 2k + 3 - [2k^2 +4] = -25$
$2k^2 - 8k - 5 -(2k^2 + 4) = -25$
$-8k-9=-25$, $-8k = -16$, $k = 2$.

FAIRFIELD COUNTY MATH LEAGUE 2015-16 Match 3 Team Round

Note: The inverse of a function or relation is not necessarily a function.

1.) _______24_______minutes 4.) _______3, 30_______

2.) _____ 4.9 x 10^{-8} ___ seconds 5.)__$(15625, 2), (\frac{1}{15625}, -1)$__

3.) __________1,4,7,A,D_______ 6.) ______$27, \frac{\sqrt{3}}{27}$________

1.) Working alone, Jeff can grade a stack of Math League papers in 10 minutes. Andrew can grade the same stack in 12 minutes . They work together for 3 minutes when Michael joins in and begins helping. They finish the task 2 minutes later. How long would it take Michael to grade the stack of papers alone?. Give your answer in minutes.

Jeff can grade $\frac{1}{10}$ of the pile in one minute, and Thai can grade $\frac{1}{12}$ of the pile in one minute, and they both work for 3 minutes, so they grade $\frac{3}{12}+\frac{3}{10}=\frac{11}{20}$ of the pile. That leaves $\frac{9}{20}$ to grade, and if Michael takes x minutes to grade one pile, then $\frac{2}{12}+\frac{2}{10}+\frac{2}{x}=\frac{9}{20}$. Multiply by the common denominator 60x to get

$10x+12x+120=27x$

$22x+120=27x$

$5x=120$

$x=24$

2.) In the hexadecimal system, A=10, B=11, C=12, D=13, E=14, and F=15. Ax and xD are two-digit hexadecimal numbers. Find all hexadecimal digits x such that the expression Ax+xD-x^2 is divisible by 3.

If x is divisible by 3, so is x^2. If x is not divisible by 3, neither is x^2, so it is easier to look at Ax+xD-x: Ax-x is always 160. xD is x*16+13, so Ax+xD-x is for the digits 0,1,2,3,...D,E,F: 173, 189, 205,221, 237,...413. Keep adding 16 every time and since A1+1D -1 is divisible by 3, every time you add 48 you will again get something divisible by 3, so that happens every third digit starting from 1, so 1,4,7,A,D.

3.) Spontaneous fission of a uranium nucleus which is at rest occurs. An alpha particle is emitted to the east at 2.0000×10^7 meters/second. The heavier thorium nucleus recoils to the west at 3.4×10^5 meters/second. After how many seconds will the two particles be one meter away from each other? Give your answer in scientific notation, rounded to one decimal place.

The particles go different ways, so since the times are the same, we have $(2.0000\text{x}10^7)t + (3.42 \text{ x } 10^5)t = 1$, so $t = \frac{1}{2.034x10^7} = \frac{1}{2.034}x10^{-7}$. Long division gives $\frac{1}{2.034} \approx 0.49$,

so

$4.9x10^{-1}x10^{-7} = 4.9x10^{-8}$ seconds.

4.) The degree measure of the sum of the interior angles of a convex n-gon is 180 more than 12 times the number of its diagonals. Find all possible values of n.

$$180(n-2) = 12 * \frac{n(n-3)}{2} + 180$$

$$180(n-2) = 6n(n-3) + 180$$

$$30(n-2) = n(n-3) + 30$$

$$30n - 60 = n^2 - 3n + 30$$

$$n^2 - 33n + 90 = 0$$

$$(n-3)(n-30) = 0$$

$$n = 3\_or\_n = 30$$

5.) If f(x)= 0.2^{x^2-5x} and g(x) = 25^{2x-1}, give the coordinates of all points where the relations $f^{-1}(x)$ and $g^{-1}(x)$ intersect.

$$f(x) = 0.2^{x^2-5x} = 5^{-1(x^2-5x)} = 5^{-x^2+5x}$$

$$g(x) = 25^{2x-1} = 5^{2(2x-1)} = 5^{4x-2}$$

$$Set\_-x^2 + 5x = 4x - 2,$$

$$-x^2 + x + 2 = 0$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2\_or\_x = -1$$

$$If\_x = 2, f(x) = g(x) = 25^{2*2-1} = 25^3 = 15625$$

$$If\_x = -1, f(x) = g(x) = 25^{2(-1)-1} = 25^{-3} = \frac{1}{15625}$$

Since this is where f(x) and g(x) intersect, if you interchange the coordinates you get the ordered pairs where $f^{-1}(x)$ and $g^{-1}(x)$ intersect, so the coordinates are $(\frac{1}{15625}, -1), (15625, 2)$

6.) Find all values of x such that the matrix $\begin{bmatrix} 2 & 5 \\ \log_x 27 & \log_3 x \end{bmatrix}$ has determinant 1. Express your answers in simplest radical form if necessary.

$\log_x 27 = \frac{\log_3 27}{\log_3 x} = \frac{3}{\log_3 x}$, so the determinant is

$$2 * \log_3 x - \frac{3}{\log_3 x} * 5$$

$$Solve\_2 * \log_3 x - \frac{3}{\log_3 x} * 5 = 1$$

$$Multiply\_by\_\log_3 x$$

$$2(\log_3 x)^2 - 15 = \log_3 x$$

$$2(\log_3 x)^2 - \log_3 x - 15 = 0$$

$$(2\log_3 x + 5)(\log_3 x - 3) = 0$$

$$\log_3 x = \frac{-5}{2}\_or\_\log_3 x = 3$$

$$x = 3^{-\frac{5}{2}} = \frac{1}{3^{\frac{5}{2}}} = \frac{1}{9 * 3^{\frac{1}{2}}} = \frac{1}{9\sqrt{3}} = \frac{\sqrt{3}}{9\sqrt{3}\sqrt{3}} = \frac{\sqrt{3}}{27}$$

$$or\_x = 3^3 = 27$$