2014–2015

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 2 Round 1
Arithmetic: Factors
And Multiples

1) 28

2.) 3937

3.) 300, 900, 2700

1) Find the sum of all of the prime factors of 2310.
2310 = 2*3*5*7*11, so 2+3+5+7+11 = 28

2) If M can take on any integer value from 1 through 10, find the least common multiple of all numbers of the form $2^M - 1$ which are not divisible by either 3 or 7.
$2^1 - 1 = 1$, which doesn't factor in; $2^2 - 1 = 3$ which is divisible by 3, $2^3 - 1 = 7$ which is divisible by 7, $2^4 - 1 = 15$, which is divisible by 3, $2^5 - 1 = 31$, which is divisible by neither 3 nor 7, $2^6 - 1 = 63$ which is divisible by both 3 and 7, $2^7 - 1 = 127$ which is divisible by neither 3 nor 7, $2^8 - 1 = 255$, which is divisible by 3 since 2+5+5 is divisible by 3, $2^9 - 1 = 511$, which is 7 times 73, and $2^{10} - 1 = 1023$, which is divisible by 3 since 1+0+2+3 is divisible by 3. So the answer is 31*127 = 3937

3) The greatest common factor of N and 840 is 60. The least common multiple of N and 135 is 2700. Find all possible values of N.

N must have factors of 2^2*3*5, but not 2^3 or 7 since $840=2^3*3*5*7$. 135 and 2700 both have 3^3, but the 5^2 must come from N since 135 has only one factor of 5. N could have one, two, or three factors of 3, so N could be $2^2*3*5^2 = 300$, or $2^2*3^2*5^2 = 900$, or $2^2*3^3*5^2 = 2700$

2) If N can take on any integer value from 1 through 10, find the least common multiple of all numbers of the form $2^N - 1$ which are divisible by neither 3 nor 7.

$2^1 - 1 = 1$, which doesn't factor in; $2^2 - 1 = 3$ which is divisible by 3, $2^3 - 1 = 7$ which is divisible by 7, $2^4 - 1 = 15$, which is divisible by 3, $2^5 - 1 = 31$, which is divisible by neither 3 nor 7, $2^6 - 1 = 63$ which is divisible by both 3 and 7, $2^7 - 1 = 127$ which is divisible by neither 3 nor 7, $2^8 - 1 = 255$, which is divisible by 3 since 2+5+5 is divisible by 3, $2^9 - 1 = 511$, which is 7 times 73, and $2^{10} - 1 = 1023$, which is divisible by 3 since 1+0+2+3 is divisible by 3. So the answer is 31*127 = 3937

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 2 Round 2
Algebra: Polynomials
And Factoring

1.) $12a^2b+16b^3$ or $4b(3a^2+4b^2)$_
(equivalent)

2.) ______$(8x+3y)(3x-8y)$_____

3.) __$mk^2n^3(k+2n)(m+1)(m-1)$_

1) Simplify as much as possible:

$$(a+2b)^3-(a-2b)^3$$
$$=(a+2b)(a^2+4ab+4b^2)-(a-2b)(a^2-4ab+4b^2)$$
$$=(a^3+6a^2b+12ab^2+8b^3)-(a^3-6a^2b+12ab^2-8b^3)$$
$$=12a^2b+16b^3$$

Or $4b(3a^2+4b^2)$

2) Factor into two binomials: $24x^2-55xy-24y^2$

Try different combinations: -55 = -64+9, so that's a good way to try to split the middle term

$$24x^2-55xy-24y^2$$
$$=24x^2-64xy+9xy-24y^2$$
$$=8x(3x-8y)+3y(3x-8y)$$
$$=(8x+3y)(3x-8y)$$

3) Factor as much as possible: $m^3k^3n^3-mk^3n^3+2m^3k^2n^4-2mk^2n^4$

$$m^3k^3n^3 - mk^3n^3 + 2m^3k^2n^4 - 2mk^2n^4 =$$

$$mk^2n^3(m^2k - k + 2m^2n - 2n)$$

$$= mk^2n^3(k(m^2 - 1) + 2n(m^2 - 1))$$

$$= mk^2n^3(k + 2n)(m^2 - 1)$$

$$= mk^2n^3(k + 2n)(m + 1)(m - 1)$$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 2 Round 3 Geometry: Pyth. Thm, Area, Perimeter

1)_______60_______cm

2.) ____12___________

3.) _6x+48, 15x+120 _____
(factored form OK)

1) The two legs of a right triangle differ by 14 cm. The area of the triangle is 120 cm^2. Find the perimeter of the triangle.
Let x=shorter leg, x+14=longer leg. $(1/2)x(x+14)=120$, so $x^2+14x-240=0$, so $(x+24)(x-10)=0$, so x=10. The other leg is 10+14=24. Since 10 and 24 are each 2 times the numbers 5 and 12, and 5-12-13 is a Pythagorean triple, the hypotenuse must be 2*13=26, so the perimeter is 10+24+26=60 cm.

2.) In right triangle ABC, the right angle is at C. $AB = x\sqrt{13}$, BC=4x-2, and AC=5x-6. Find the area of the triangle.
$(4x-2)^2 + (5x-6)^2=(x\sqrt{13})^2$, so $16x^2-16x+4 + 25x^2-60x+36= 13x^2$,

so $28x^2-76x+40=0$, $7x^2-19x+10=0$, so $(7x-5)(x-2)=0$. X=2 or $x=\frac{5}{7}$, but

$5*\frac{5}{7}-6$ is a negative number, so x=2. The area is $\frac{1}{2}*(4*2-2)(5*2-6)$
$= 0.5(6)(4) = 12$

so the area is 12.

3) The two bases of an isosceles trapezoid have lengths x and (x+16). The sides of the trapezoid all have whole number lengths. Find all possible values for the area of the trapezoid in terms of x. The two bases are x and x+16. The trapezoid can be split into 3 parts, 2 triangles with base 8 and height h, and the rectangle with base x and height h. Since the hypotenuse of the triangle must be a whole number, it must either be a 6-8-10 triangle or an 8-15-17 triangle. If it is a 6-8-10 triangle, the height is 6, and the area is (1/2)(x+x+16)(6)=6x+48. If it is an 8-15-17 triangle, the height is 15, and the area is (1/2)(x+x+16)(15)=15x+120

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 2 Round 4 Algebra 2: Inequalities And Absolute value

1)_____ $x<0$ _or_ $x>\frac{2}{3}$ _

2.) __________2.5, −1.5________

3.) ________ $\frac{-1}{3}<x<\frac{10}{3}$ _______

1) Find all values of x such that the expression $\frac{4x}{3x-2}$ is positive.

If 3x-2>0, then 4x>0, so x>2/3 and x>0, means $x>\frac{2}{3}$. If 3x-2<0, then 4x<0, so x<0.

2) Find all values of x such that $|x-2|=3|x-1|-4$.

If x>2, we have x-2 = 3x-3-4, so y-2=3x-7, so 2x=5, and x=2.5
If 1<x<2, we have 2-x=3x-3-4, so 2-x=3x-7, and 4x=9, so x=2.25, but that is not in this range.
If x<1, we have 2-x = 3(1-x)-4, so 2-x=3-3x-4, so 2-x=-1-3x, so 2x=-3, so x=-1.5

3) Find all values of x such that $5+|7-3x|<16-|3x-2|$

If $\frac{2}{3}<x<\frac{7}{3}$, both expressions are positive, so 5+7-3x < 16-(3x-2),

So 12-3x<18-3x always, so $\frac{2}{3}<x<\frac{7}{3}$ is one region.

If $x=\frac{2}{3}$, the statement is clearly true. If $x\le\frac{2}{3}$, this becomes 5 + (7-3x)<16-(2-3x),

So 12-3x<14+3x, so 6x>-2, so $x>\frac{-1}{3}$. If $x=\frac{7}{3}$, the statement is true. If $x>\frac{7}{3}$, then we have 5+(3x-7) < 16-(3x-2), so 3x-2<-3x+18, so 6x<20, and $x<\frac{10}{3}$. Put these all together and the answer is $\frac{-1}{3}<x<\frac{10}{3}$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 2 Round 5
Trigonometry:
Laws of Sine and Cosine

1.) ________10___________

2.) _____$2\sqrt{61}+2\sqrt{151}$_____cm

3.) _________$\sqrt{55}$__________

1.) In ΔXYZ, $\sin\angle YXZ=\frac{1}{7}$ and $\sin\angle XYZ=\frac{1}{3}$. If YZ=k and XZ=$k^2$-2, find the numerical value of YZ+XZ.

$$\frac{k}{\frac{1}{7}}=\frac{k^2-2}{\frac{1}{3}},$$

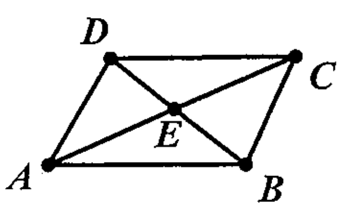
$$7k=3(k^2-2)$$

$$3k^2-7k-6=0$$

$$(3k+2)(k-3)=0,$$

$$\therefore k=3, k^2-2=7,$$

$$sum\_is\_10$$



2.)The diagonals of parallelogram ABCD have length 18 cm and 10 cm meet at E. Angle CEB measures 60 degrees. Find the perimeter of the parallelogram.

The diagonals of a parallelogram bisect each other, so CE=9 and BE=5. Since cos(angle CEB) = ½, we have cos(angle AEB) = -1/2

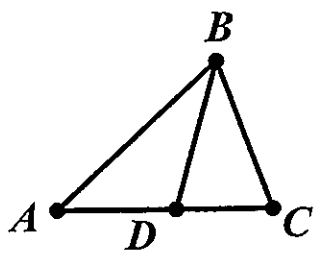
$$CB^2 = 9^2 + 5^2 - 90(\frac{1}{2})$$

By law of cosines,

$$CB^2 = 61, CB = \sqrt{61}$$

$$AB^2 = 9^2 + 5^2 - 90(\frac{-1}{2})$$

$$AB^2 = 151, AB = \sqrt{151}$$



3.) $\overline{BD}$ is a median of ΔABC. If BD=6, BC=7, and AD=4, find the length of $\overline{AB}$.

$AD = 4, so\_DC = 4.$

$7^2 = 4^2 + 6^2 - 2 * 4 * 6 * \cos\angle BDC,$

$$\cos\angle BDC = \frac{7^2 - 4^2 - 6^2}{-2 * 4 * 6} = \frac{1}{16}$$

$$\therefore \cos\angle ADB = -\frac{1}{16},$$

$$AB^2 = 6^2 + 4^2 - 2 * 4 * 6 * (-\frac{1}{16}) = 36 + 16 + 3 = 55$$

$AB = \sqrt{55}$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 2 Round 6
Writing Equations of Lines

1.) ______ $y=4x-\frac{11}{20}$ ________

2.) ________ $y=\frac{3}{4}x+\frac{31}{4}$ __

3) ________ $y=\frac{-1}{3}x+4$ __

and ______ $y=-3x+12$ ________

1.) $x=\frac{1}{6}t+\frac{1}{5}$ and $y=\frac{2}{3}t-\frac{1}{4}$ represent the parametric equations of a line. Find the equation of the line in the form y=mx+b.

$$x=\frac{1}{6}t+\frac{1}{5},\frac{1}{6}t=x-\frac{1}{5},5t=30x-6,t=6x-\frac{6}{5}$$

$$y=\frac{2}{3}(6x-\frac{6}{5})+\frac{1}{4},y=4x-\frac{2}{3}*\frac{6}{5}+\frac{1}{4},$$

$$y=4x-\frac{4}{5}+\frac{1}{4},y=4x-\frac{16}{20}+\frac{5}{20},$$

$$y=4x-\frac{11}{20}$$

2.) A circle of radius 5 is centered at (2,3). A radius of the circle lies on the line 4x+3y=17 and intersects the circle at a point in the second quadrant. Find the equation of the tangent line to the circle through this point. Express your answer in the form y=mx+b.

The given line has slope $\frac{-4}{3}$, and since the circle has radius 5, we can find a point on the circle by either going down 4 and over 3, or up 4 and back 3. The second option gives us a point in the second quadrant (-1, 7). The tangent line is perpendicular to the radius so its slope is $\frac{3}{4}$, so y-7=$\frac{3}{4}$(x+1), y=$\frac{3}{4}$x+7+$\frac{3}{4}$, $y=\frac{3}{4}x+\frac{31}{4}$

3) The length of the segment connecting the points (1,x) and (x,5) is $\sqrt{10}$. Find the two possible equations for the perpendicular bisector of the segment. Express your answers in the form y=mx+b.

$$\sqrt{(x-1)^2+(5-x)^2}=\sqrt{10}$$

$$\sqrt{(x^2-2x+1+25-10x+x^2)}=\sqrt{10}$$

$$2x^2-12x+26=10$$

$$2x^2-12x+16=0$$

$$x^2-6x+8=0, \therefore x=2, x=4$$

The midpoint of (1,2) and (2,5) is (1.5, 3.5) and the slope of the line is 3. Its perpendicular bisector has slope $\frac{-1}{3}$. The line has equation

$$y-\frac{7}{2}=\frac{-1}{3}(x-\frac{3}{2})$$

$$y=\frac{-1}{3}x+4$$

The midpoint of (1,4) and (4,5) is (2.5,4.5), and the slope of its line is $\frac{1}{3}$, so the slope of the perpendicular bisector has slope -3. Its equation is

$$y-\frac{9}{2}=-3(x-\frac{5}{2})$$

$$y=-3x+12$$

FAIRFIELD COUNTY MATH LEAGUE 2014-15 Match 2 Team Round

1.) ________12(2x+5)(x+7)_____ 4.) ____________ $8-2\sqrt{3}$ __________

2.) ________ $z\le -2$ or $2\le z\le 3$ ____ 5.) ______________20___________

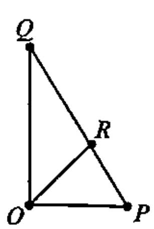
3.) ___________2, -4_________ 6.) __________ $y=\frac{5}{3}x-\frac{34\sqrt{2}}{3}$ _______

1.) A quadratic polynomial has form $Dx^2 + Ex + F$. D is the greatest common factor of 96 and 216. E is the least common multiple of 57 and 76. F is the smallest natural number that is evenly divisible by 2,3,4,5,6, and 7.
Give the complete factoring of $Dx^2 + Ex + F$ with natural number coefficients.

$96=2^5*3$, $216=2^3*3^3$, so the GCF is $2^3*3 = 24$. 57 = 19*3, 76=19*4, so the GCF is 19*3*4=228. 420 is the smallest number contains 2^2, 3, 5, and 7. So the polynomial is $24x^2 + 228x + 420 = 12(2x^2 + 19x + 35)=12(2x+5)(x+7)$

2.) Find all values of z such that $-z^3+3z^2+4z-9\ge 3$
$-z^3+3z^2+4z-12\ge 0$, so $z^3-3z^2-4z+12\le 0$. The left hand side factors by grouping to (z-3)(z+2)(z-2). This is negative when only one of the terms is negative, or when all three terms are negative. When $2\le z\le 3$, z-3 is the only nonpositive term. When $z\le -2$, all three terms are nonpositive. So the answer is $z\le -2$ or $2\le z\le 3$..

3.) Find all integers m such that $|m+2|+|m-1|=|m^2-9|$
If x>3, all 3 expressions are positive, so $(x+2)+(x-1)=x^2-9$, so $x^2-2x-10=0$, which does not factor so no integer values. If 1<x<3, we have $(x+2)+(x-1)=9-x^2$, so $x^2+2x-8=0$, so (x+4)(x-2)=0, so in this range x=2. If -2<x<1, we have $(x+2)+(1-x)=9-x^2$, so $x^2-6=0$, no integer values in this range. If -3<x<-2, we have $(-x-2)+(1-x)=9-x^2$, so $x^2-2x-10=0$ again, and no integer values. If x<-3, we have $(-x-2)+(1-x)=x^2-9$, which again simplifies to $x^2+2x-8=0$, and this time x=-4 is the solution we want.



4.) In right triangle OPQ, the right angle is at O, O has coordinates (2,2), P has coordinates (6,2), $\angle OPQ = 60°$, and $\overline{OR}$ is the angle bisector of $\angle POQ$. If R lies on $\overline{PQ}$, find the x-coordinate of point R.

Since P is at (2,2), O is a right angle with one ray horizontal and the other vertical, and segment OR is an angle bisector, OR must be along the line y=x, so x and y have the same coordinates. Since angle P is 60 degrees and the length of segment OP is 4, the length of OQ must be $4\sqrt{3}$, so the coordinates of Q are $(2, 2+4\sqrt{3})$.

The slope of line PQ is $\frac{(2+4\sqrt{3})-2}{2-6} = -\sqrt{3}$, so the equation of line PQ is

$y-2 = -\sqrt{3}(x-6)$. We want

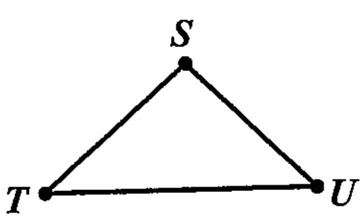
$$x-2 = -\sqrt{3}(x-6)$$

$$x + x\sqrt{3} = 6\sqrt{3}+2$$

$$x(1+\sqrt{3}) = 6\sqrt{3}+2$$

$$x = \frac{6\sqrt{3}+2}{1+\sqrt{3}} = \frac{(6\sqrt{3}+2)(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} = \frac{6\sqrt{3}-18+2-2\sqrt{3}}{-2} =$$

$$\frac{-16+4\sqrt{3}}{-2} = 8-2\sqrt{3}$$

5.) 

5.) In ΔSTU, SU=6, ST=x, TU=x+2.
The cosine of $\angle SUT$ is six times the cosine of $\angle TSU$. Find the perimeter of ΔSTU.

We have $(x+2)^2 = x^2 + 36 - 12x\cos(\text{angle TSU})$, so cos(angle TSU)=

$$\frac{(x+2)^2 - x^2 - 36}{-12x} = \frac{4x-32}{-12x}$$

We have $x^2 = (x+2)^2 + 36 - 12x(\cos(\text{angle SUT})$, so cos(angle SUT) =

$$\frac{x^2-(x+2)^2-36}{-12(x+2)} = \frac{-4x-40}{-12(x+2)}$$

$$6*\frac{4x-32}{-12x}=\frac{-4x-40}{-12(x+2)}$$

Simplify

$$6*\frac{x-8}{x}=\frac{-x-10}{x+2}$$

$$6(x+2)(x-8)=-x(x+10)$$

$$6x^2-36x-96=-x^2-10x$$

$$7x^2-26x-96=0$$

$$(7x+16)(x-6)=0$$

$$x=6$$

So the perimeter is 6+6+8 = 20

6.) Line m has equation $x+y=10\sqrt{2}$ and line n has equation $x-y=2\sqrt{2}$. The two lines intersect at point A. A third line intersects line m at point B and intersects line n at point C. The x-coordinates of B and C are each greater than the x-coordinate of A, the area of ΔABC is 32, and the length of $\overline{AC}$ is four times the length of $\overline{AB}$. Give the equation of $\overleftrightarrow{BC}$ in slope-intercept form.

Solving $x+y=10\sqrt{2}$ and $x-y=2\sqrt{2}$ gives $x=6\sqrt{2}, y=4\sqrt{2}$ for point A. Since the slopes of the two lines are 1 and -1, A forms the right angle of right ΔABC. The area then is ½*AB*AC=32, so AB*AC=64, and AC=4*AB, then AC=16 and AB=4. Going to the right to make sure the x-coordinate is greater, start at $(6\sqrt{2},4\sqrt{2})$ and go up 16 units along $x-y=2\sqrt{2}$, which has slope 1, so you need to go $8\sqrt{2}$ units right and $8\sqrt{2}$ units up, so point C is $(14\sqrt{2},12\sqrt{2})$. Then go 4 units along the line $x+y=10\sqrt{2}$ from $(6\sqrt{2},4\sqrt{2})$. The line has slope -1, so go over $2\sqrt{2}$ and down $2\sqrt{2}$ to end up at $(8\sqrt{2},2\sqrt{2})$. The line passes through $(14\sqrt{2},12\sqrt{2})$ and $(8\sqrt{2},2\sqrt{2})$. The slope of the line is then $\frac{5}{3}$, so

$$y-2\sqrt{2}=\frac{5}{3}(x-8\sqrt{2})$$

$$y=\frac{5}{3}x-\frac{5}{3}*8\sqrt{2}+2\sqrt{2}$$

$$y=\frac{5}{3}x-\frac{40\sqrt{2}}{3}+\frac{6\sqrt{2}}{3}$$

$$y=\frac{5}{3}x-\frac{34\sqrt{2}}{3}$$