CSAML 2019

April 4, 2019
Round 1: Arithmetic and Number Theory

1. (1 point) Find the units digit in the number represented by 2019^{2019}.

2. (2 points) If $2^x \cdot 5^y \cdot 3^z = 28,800,000$, evaluate $x+y+z$.

3. (3 points) For what value of n does $1!\cdot 1+2!\cdot 2+3!\cdot 3+\cdots+n!\cdot n=2019!-1$?

Answers

1) ______________________________

2) ______________________________

3) ______________________________

CSAML 2019

April 4, 2019
Round II: Algebra I, (Real numbers and no transcendental functions)

1.(1 point) Solve for all real values of y that satisfy the condition
$$6y+7>8y+2>4y+1.$$

2. (2 points) Find all ordered pairs of integers (x, y) that satisfy all 4 conditions:

1) $-8<x<3$
2) $-10<y<-2$
3) $|x-y|=2$
4) $|x+y|=10$

3. (3 points). Find all values of x that satisfy $\left|1-\left|1-\left|1-\left|1-x\right|\right|\right|\right|=0$.

Answers

1) ______________________________

2) ______________________________

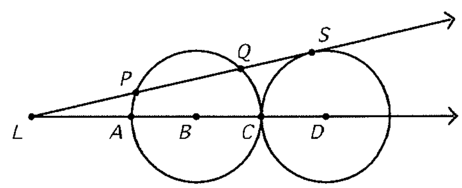
3) ______________________________

CSAML 2019

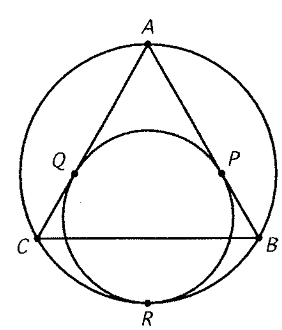
April 4, 2019
Round III: Geometry (figures are not to scale)

1. (1 point). Trapezoid $ABCD$ has bases $\overline{AB}$ and $\overline{CD}$, and $m\angle B = 112^\circ$ and $m\angle BCA = 33^\circ$. Line segments $\overline{AC}$ and $\overline{AD}$ have equal length. Find $m\angle DAC$.

2. (2 points) The diagram shows circles ω_B and ω_D externally tangent at C. The centers of the circles are B and D respectively, and each circle has radius 1. Ray $\overrightarrow{LD}$ passes through B and its other intersection point with ω_B is A. Ray $\overrightarrow{LS}$ is tangent to ω_D at S and intersects ω_B at P and Q. If $LA = 1.5$, find the distance PQ.



3. (3 points) Equilateral ΔABC of side length 6 is inscribed in circle ω. A smaller circle is internally tangent to ω at R and is tangent to $\overline{AB}$ and $\overline{AC}$ at P and Q, respectively. Find the distance PQ.



Answers
1) ______________________________

2) ______________________________

3) ______________________________

CSAML 2019

April 4, 2019
Round IV: Algebra II

1. (1 point) Solve for $x \in \mathbb{R}$: $\log_4\left(\log_{\frac{1}{4}}(x)\right) = \frac{1}{2}$.

2. (2 points) Solve for $x \in \mathbb{R}$: $x^{\frac{3}{5}} + 2x^{\frac{2}{5}} = 9x^{\frac{1}{5}} + 18$.

3.(3 points) If a and b are positive numbers that satisfy $a + \frac{1}{b} = 9$ and $b + \frac{4}{a} = 1$ then what is the value of ab?

Answers
1) ______________________________

2) ______________________________

3) ______________________________

CSAML 2019

April 4, 2019
Round V: Analytic Geometry

1. (1 point) If the distance between (1, 3) and $(a, 2a)$ is 1 unit, find all possible values of a.

2. (2 points) Find the possible values of p such that the equation $x^2 + y^2 - 6x + py + 57 = 0$ has exactly one solution (x, y).

3. (3 points) A rectangle with sides parallel to the axes is inscribed in the ellipse with minor axis endpoints (5, 8) and (5, 2) and major axis endpoint (-1, 5). If each focus of the ellipse lies on a side of the rectangle, find the area of the rectangle.

Answers

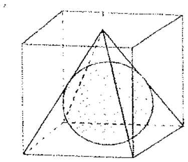
1) ______________________________

2) ______________________________

3) ______________________________

CSAML 2019

April 4, 2019
Round VI: Trigonometry, Complex Numbers



1. (1 point) Toby walks 5 miles to the north and then walks 4 miles to the southwest. He is now d miles from his starting point. Find d^2.

2. (2 points) Lines L_1 and L_2 both have positive slope. The slope of L_2 is four times that of L_1. The angle L_2 makes with the positive x-axis is twice the equivalent angle for L_1.
Find the slope of L_1.

3. (3 points) PQR is a right triangle with right angle at P. Points A and B lie on QR with $QA = RB = \frac{QR}{4}$. Furthermore, there is an acute angle θ with $\sin\theta = PA$ and $\cos\theta = PB$. Find the length QR.

Answers

1) ____________________________

2) ____________________________

3) ____________________________

CSAML 2019

April 4, 2019 TEAM ROUND

1) For what positive integer value(s) of n is $n^2 - 132$ a perfect square?

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2) Let $\lfloor x \rfloor$ represent the greatest integer less than or equal to x and let $\{x\}$ represent the fractional part of x, $x - \lfloor x \rfloor$. Let $y = \dfrac{33\{\sqrt{2}\} + 62}{\{\sqrt{2}\} + 4}$. Find $\lfloor y \rfloor$.

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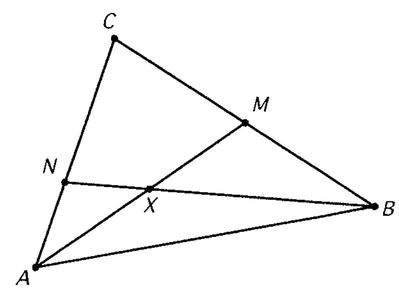
3) In ΔABC, M is the midpoint of side $\overline{BC}$ and point N divides side $\overline{AC}$ in a $2:3$ ratio. Line segments $\overline{AM}$ and $\overline{BN}$ intersect at X.

Compute the quantity $\dfrac{NX}{NB}$.



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4). If $x^4 + ax^2 + bx - 36 = 0$ has 4 distinct integer solutions for x, find all possible ordered pairs (a, b).

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5) Consider 2 circles in the xy-plane, one with equation $x^2 + y^2 - 1 = 0$ and the other with equation $x^2 + y^2 + x - \sqrt{3}y + k = 0$. The circles have centers C and M and intersect each other at points S and L. If $0 < k < 1$ and the area of quadrilateral $CSML$ is $\dfrac{1}{2}$, find the value of k.

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6) Note that, by De Moivre's theorem

$$\sum_{k=0}^{\infty} \frac{1}{2^k}(\cos\theta + i\sin\theta)^k = \sum_{k=0}^{\infty} \frac{1}{2^k}(\cos k\theta + i\sin k\theta).$$

Compute the sum $\displaystyle\sum_{k=0}^{\infty} \frac{1}{2^k}\cos\frac{k\pi}{3}$.
