NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2010

Round 1: Arithmetic and Number Theory

1. _____

2. _____

3. _____

1. How many 2-digit prime numbers have the property that the sum of its digits is divisible by 5?

2. Determine the largest possible prime number that can be expressed by $x^2 + 30x - 175$, where x is an integer.

3. Let x be the two-digit number AB, $A \neq 0$. Let y be the six-digit number $ABABAB$.

   For those values of x for which $\frac{y}{x^2}$ is an integer, determine all prime numbers that could be factors of $\frac{y}{x^2}$.

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Round 2: Algebra 1

1. _____

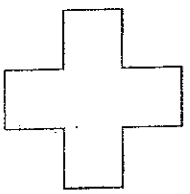
2. _____

3. _____

1. Determine the smaller value of $\frac{x}{y}$ given $2x^2 + 2xy = 3xy + 3y^2$.

2. Solve for all values of x such that: $x^4 - x^3 - 6x^2 + 6x = x^3 - x^2$.

3. In the figure, all sides are congruent and the angle between each pair of consecutive sides is 90°. If the numerical value of the perimeter subtracted from the numerical value of the area equals K, determine the least possible value of K.



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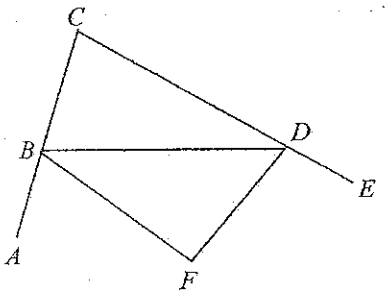
Round 3: Geometry

1. _____

2. _____

3. _____

1. In $\Delta ABCD$, $\overline{BF}$ and $\overline{DF}$ are the trisectors of $\angle ABD$ and $\angle EDB$ respectively such that $m\angle DBF = \frac{1}{3}m\angle DBA$ and $m\angle BDF = \frac{1}{3}m\angle BDE$. If $m\angle C = 78°$, determine $m\angle F$.



2. ABC is an equilateral triangle of side 6. Circle P is tangent to $\overline{BC}$ at D and passes through the trisection points of $\overline{AC}$. Find the length of $\overline{CD}$.

3. In ΔABC, $m\angle C = 90°$ and $BC = 7$. Point D lies on $\overline{AB}$ such that $BD = 10$ and $AD = DC$. Find the length of $\overline{AC}$.

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

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Round 4: Algebra 2

1. _____

2. _____

3. _____

1. If $\log_3 y = 2\log_5 x$, then $y = x^k$. If k can be written as $\log_a b$, find the ordered pair (a, b), where a and b are integers and $a + b$ has the smallest possible sum.

2. If $4^{1/x} - 8^{1/y} = 0$ and $\log_2 x - \log_4 y = 0$, find all ordered pairs (x, y).

3. Let $f(x) = ax^3 + bx^2 + cx + d$. If $f(1) = 1$, $f(2) = 2$, and $f(3) = 3$, determine the value of $\frac{b}{d}$.

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Round 5: Analytic Geometry

1. _____

2. _____

3. _____

1. The domain of a relation is $-3 \le x \le 4$ and its range is $-1 \le y \le 8$. If the graph of the relation is rotated through 90° counterclockwise relative to the origin, determine the domain of the rotated graph.

2. Determine the length of the shortest path from the origin to $A(8,6)$ that does not go inside the region determined by the quadrilateral $MNPQ$ given $M(2,4), N(5,4), P(5,1), Q(2,1)$.

3. Points $A(-4, 2)$, $B(4, 2)$, $C(4, -2)$, and $D(-4, -2)$ lie on the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b$.

   If $\overline{AD}$ and $\overline{BC}$ pass through the focal points of the ellipse, determine the value of $a^2 + b^2$.

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Round 6: Trig and Complex Numbers

1. _____

2. _____

3. _____

1. Find all real values of a for which $\cos x = -\frac{1}{a}$ and $\cos\frac{x}{2} = \frac{1}{\sqrt{a}}$.

2. For $k > m > 0$, the tangent of the acute angle bounded by $y = kx$ and $y = mx$ is twice the tangent of the first quadrant angle bounded $y = mx$ and the x-axis. If k and m are reciprocals, find k.

3. For $k > 0$ and $i = \sqrt{-1}$, let $a_1 = 1$, $a_n = \frac{i}{k}a_{n-1}$, $b_1 = -1$, and $b_n = -\frac{i}{k}b_{n-1}$. If $\sum_{i=1}^{\infty} a_i - \sum_{i=1}^{\infty} b_i = \frac{16}{9}$, determine the value of k.

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

NEW ENGLAND PLAYOFFS – 2010

Team Round

1. _____ **4.** _____

2. _____ **5.** _____

3. _(___ ,____,____,____)_ **6.** _____

1. For $a, b > 0$, if $\sin\left(\tan^{-1}\frac{a}{b}\right) = \frac{b}{a}$, determine the value of $\frac{a^2}{b^2}$.

2. Let f be a function defined for all real numbers and let f have the property that $f(3 - x) = f(x + 5)$ for all x. If f has seven distinct roots, determine the average of the roots.

3. Let $P(x)$ be a cubic polynomial whose coefficients are all positive integers. $P(1) = 11$ and $P(P(1)) = 5701$. If $P(x) = ax^3 + bx^2 + cx + d$, determine the ordered quadruple (a, b, c, d).

4. Given weights of 1, 2, 3, 4, 5, and 6 pounds, two are selected at random and placed on one side of a scale. From the remaining four, two are selected at random and placed on the other side of the scale. What is the probability that the scale balances?

5. In ΔABC, $m\angle C = 84°$, $m\angle B = 54°$, $BC = a$ and $AC = b$, where a and b are lengths appropriate for a triangle with those angles.
Determine the length of $\overline{AB}$ solely in terms of a and b.

6. Points A and B lie on circle O of radius 12 such that $m\angle AOB = 120°$. Point P lies on minor arc(AB) such that $m\angle POB = 40°$. Perpendiculars from P intersect $\overline{OA}$ and $\overline{OB}$ at C and D respectively. Determine the length of $\overline{CD}$.

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

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Answer Sheet

Round 1

1. 5
2. 41
3. 3, 7, 13, 37

Round 2

1. -1
2. $\{0, 1, 3, -2\}$
3. $-\frac{36}{5}$

Round 3

1. 94
2. $2\sqrt{2}$
3. $3\sqrt{39}$

Round 4

1. $(5, 9)$
2. $\left(\frac{3}{2}, \frac{9}{4}\right)$
3. 1

Round 5

1. $-8 \le x \le 1$
2. $2\sqrt{5} + 2\sqrt{10}$
3. $20 + 4\sqrt{17}$

Round 6

1. 3
2. $\sqrt{5}$
3. $2\sqrt{2}$

Team

1. $\frac{1+\sqrt{5}}{2}$
2. 4
3. $\{4, 3, 1, 3)$
4. $\frac{7}{45}$
5. $\sqrt{a(a+b)}$
6. $6\sqrt{3}$