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Match 6 Round 1
Geometry: Lines and Angles

Note: Figures not necessarily Drawn to scale

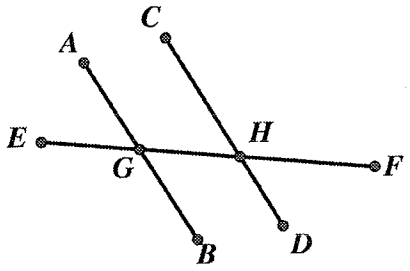
1.) 90 degrees

2.) 142 degrees

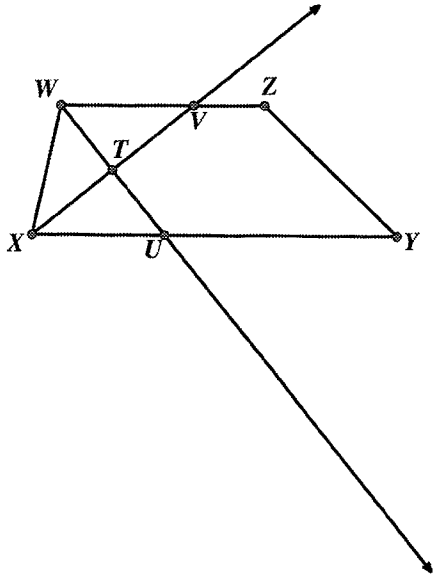
3.) 94 degrees

1.)_ A triangle is formed by connecting points A(-2,1), B(1,3), and C(5,-3). What is the sum of the measures of $\angle BAC$ and $\angle BCA$?

2.)_ $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$. The lines are cut by transversal $\overrightarrow{EF}$, which intersects line $\overrightarrow{AB}$ at G and $\overrightarrow{CD}$ at H. If $m(\angle HGB) = (31+3x)^\circ$ $m(\angle CHF) = (6x+29)^\circ$, find $m(\angle AGE) + m(\angle FHD)$.



3.) In trapezoid WXYZ, $\overline{WZ}$ is parallel to $\overline{XY}$, $\overline{XV}$ bisects $\angle WXY$ and $\overline{WU}$ bisects $\angle XWZ$. The bisectors meet at T. $\angle WXT = (6x-5)^\circ$ and $\angle XWT = (x^2-17)^\circ$. Find the measure of $\angle ZWX$



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Match 6 Round 2
Algebra: Literal Equations

1.)_ z= $3x+4y-5$

2.) $R_3 = \frac{R R_1 R_2}{R_1 R_2 - R R_1 - R R_2}$

3.) __y = $x+2$

1.)_ Solve for z in terms of x and y:

$$3x+5y-z=15+2z-6x-7y$$

2.)_ If three resistors with resistance R_1, R_2, and R_3 are arranged in an electric circuit in parallel, the formula for the equivalent resistance R is found by the formula $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$. Solve this equation for R_3 in terms of R, R_1, and R_2. Express your answer as a single fraction.

3.)_ If $x \neq 2$, solve for y in terms of x:

$$x^4-x^3y-2yx+8y+x^2y=2yx-x^2y+16$$

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Match 6 Round 3
Geometry:
Solids and
Volumes

1.) $\frac{72}{\pi}$ in^3

2.) $288\pi - 192\sqrt{3}$ cm^3

3.) k= $2\sqrt[3]{4}$

1._ The lateral area of a cylinder (the surface area not including the bases) with height 8 inches is 48 in^2. What is the volume of the cylinder?

2. A cube is inscribed in a sphere of radius 6 cm. What volume is inside the sphere but outside the cube?

3. A cone is formed by rotating the line $y = \frac{2}{5}x$ from x=0 to x=10 around the y-axis. The plane y=k cuts the cone so that half of its volume is above the plane y=k. Find the value of k.

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Match 6 Round 4
Radical Expressions and Equations

1.) $\frac{29\sqrt{5}}{5}$

2.) $\sqrt[6]{1125}$

3.) 4

1.)_ Express as a single reduced fraction in simplest radical form:

$$3\sqrt{45}+\frac{4}{\sqrt{5}}-\sqrt{80}$$

2) Express as a single radical: $\sqrt[3]{3}\sqrt{5}$

3._Solve for all real values of x:

$$\sqrt{2x+1}-\sqrt{x-3}=\sqrt{5x-16}$$

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Match 6 Round 5
Polynomials and
Advanced
Factoring

1.) 144

2.) -65, 7, 135

3.) 7

1.) What is the remainder when $x^5 - 4x^3 + x^2 - x + 3$ is divided by $x - 3$?

2.) $x^3 + ax^2 + bx + 7$ factors into three binomials with integer coefficients. Find all possible values of ab.

3) A quartic polynomial $x^4 + ax^3 + bx^2 + cx + d$ where a, b, c, and d are integers has 1+i and 3-2i as two of its zeros. Find a+b+c+d.

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Match 6 Round 6
Counting and
Probability

1.) 4536

2.) $\frac{5}{12}$

3.) (1,0), (3,1), (8,2)

1.)_ How many 4 digit numbers N such that $1000 \leq N \leq 9999$ have no repeating digits? (e.g., 4576 has no repeating digits, but 4546 has a repeating 4).

2.)_ The names of five boys and five girls are placed into a hat and three names are drawn out without replacement. What is the probability that the names of exactly two girls were drawn?

3.) ${}_N P_R$ is the number of permutations of N objects taken R at a time. ${}_N C_R$ is the number of combinations of N objects taken R at a time. If $1 \leq N \leq 10$, find all ordered pairs (N,R) such that ${}_N P_R = {}_N C_{R+1}$

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Match 6
Team
Round

1.) _E = 9A - 1470

2.) 3, 66, 731

3.) $8A\sqrt{3A}$

4.) $(a-2)(a-4)(a^2+5)$

5.) $\frac{64}{81}$

6.) M = 12 N = 20

1.)_ Angles A, B, C, D, and E are the five interior angles of a convex pentagon. The measure of $\angle B$ is equal to ten times the measure of the supplement of $\angle A$. Two times the measure of $\angle B$ is eighty degrees more than the measure of $\angle C$. The measure of $\angle D$ is thirty degrees more than the supplement of the measure of $\angle C$. Find the measure of $\angle E$ in terms of measure of $\angle A$. Use E for the measure of $\angle E$ and A for the measure of $\angle A$.

2.) Find all values of x such that $\sqrt{x-2} - 6\sqrt[3]{x-2} + 11\sqrt[6]{x-2} = 6$

3). A tetrahedron has surface area $24A\sqrt{3}$. Give the volume of the tetrahedron in terms of A in simplest radical form.

4.) Factor into three binomials: $a^4 - 6a^3 + 13a^2 - 30a + 40$

5) Naphesa Collier of the UConn women's basketball team makes $\frac{2}{3}$ of her shots. If the shots are independent and she takes 5 shots in a game, what is the probability that she makes at least 3 of the 5 shots?

6) A semi-regular polyhedron is composed of M pentagons and N hexagons. An edge of the polyhedron is formed when one side of one of the polygons meets a side of another polygon. A vertex of the polyhedron is formed when one corner of a pentagon meets corners of two hexagons. The polyhedron has 90 edges and 60 vertices. Find M and N.