FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 6 Round 1
Geometry: Lines and Angles

1.) __________28______________

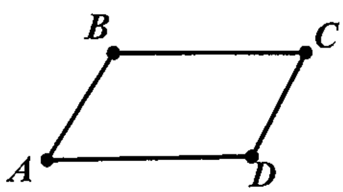
2.) ____________~~153~~ 148.5________

Note: Figures not necessarily
Drawn to scale

3.) __________109.8, 129__________

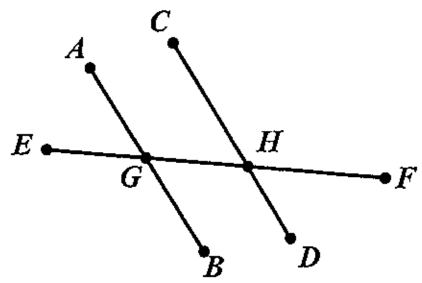
1.)_Angle J is complementary to angle K. Twice the degree measure of angle J is 6 less than the degree measure of angle K. Find the degree measure of angle J.
Let J = degree measure of angle J, K=90-J = degree measure of angle K
2J=(90-J)-6
3J=84
J=28

2.)_ In parallelogram ABCD, twice the degree measure of angle A plus 5 less than three times the degree measure of angle C is 4 more than the degree measure of angle B. Find the measure of angle D.



Opposite angles of a parallelogram are congruent, so the measures of angle A and angle C are equal. Call this value x. . Consecutive angles of a parallelogram are supplementary, so call the measure of angle B 180-x.
(2x) + (3x-5) = (180-x)+4, so 6x-5= 184-x, so 7x = 189, and x=27. The degree measure of angle D is 180-27 = 153 degrees.

3.) $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$. The lines are cut by transversal $\overleftrightarrow{EF}$, which intersects line $\overleftrightarrow{AB}$ at G and $\overleftrightarrow{CD}$ at H. If $m\angle AGE = (67-8x)$ degrees and $m\angle CHF = (5x^2+109)$ degrees, find all possible values of $m\angle CHF$ in degrees.



Angles AGE and CHF must be supplementary, since angle AGE is supplementary to angle AGF, and angles AGF and CHF are corresponding angles for two parallel lines cut by a transversal. So $(67-8x) + (5x^2 + 109) = 180$.

$5x^2 - 8x + 176 = 180$

$5x^2 - 8x - 4 = 0$

$(5x+2)(x-2) = 0$, so $x=-0.4$ or $x=2$.

If $x=-0.4$, angle CHF is $5(-0.4)^2 + 109 = 109.8$ degrees

If $x=2$, angle CHF is $5*2^2 + 109 = 129$ degrees

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Match 6 Round 2
Algebra: Literal
Equations

1.) __ $x = \dfrac{2b-3d}{a-4c}$ ____________

2.) _ $y = -k-m$ ______________

3.) ____ $z = \dfrac{e-1}{2e}, -\dfrac{1}{2}$ ______________

1.) If $a \neq 4c$, solve the equation for x in terms of the other variables:

ax-2b = 4cx – 3d
ax-4cx= 2b-3d
x(a-4c)=2b-3d

$$x = \frac{2b-3d}{a-4c}$$

2.)_ If $p \neq 2$ and $m \neq k$, solve for y in terms of k, m, and p

$$kpy + 2my + k^2p + 2m^2 = mpy + 2ky + m^2p + 2k^2$$

Rewrite this as

$$kpy + 2my - mpy - 2ky = m^2p - k^2p + 2k^2 - 2m^2$$

Factor out a GCF of y, then factor by grouping

$$y(kp + 2m - mp - 2k) = p(m^2 - k^2) - 2(m^2 - k^2)$$

$$y(p(k-m) - 2(k-m) = (p-2)(m+k)(m-k)$$

$$y(p-2)(k-m) = (p-2)(m+k)(m-k)$$

(p-2)'s cancel, and (k-m) and (m-k) cancel to -1.

$$y = -k-m$$

3.) If $e \neq 0$, solve for z in terms of e: $5ez^2 + z + 1 = ez^2 + e - z$

$$5ez^2 + z + 1 = ez^2 + e - z$$

$$4ez^2 + 2z + (1 - e) = 0$$

$$z = \frac{-2 \pm \sqrt{2^2 - 4 * 4e(1-e)}}{8e} = \frac{-2 \pm \sqrt{16e^2 - 16e + 4}}{8e} =$$

$$\frac{-2 \pm (4e - 2)}{8e}$$

$$z = \frac{4e - 4}{8e} = \frac{e - 1}{2e} \_or\_ z = \frac{-4e}{8e} = -\frac{1}{2}$$

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Match 6 Round 3 Geometry: Solids and Volumes

1.) ____ 144π ______ in^2_

2.)_______ 96π ______ in^3

3.)_ $5832 - 1458\pi$ _____ in^3_

1) A sphere has volume 288π cubic inches. What is the surface area of the sphere in square inches?

$$\frac{4}{3}\pi r^3 = 288\pi$$

$$r^3 = \frac{288 * 3}{4} = 216$$

$$r = 6$$

Surface area is $4\pi(6)^2 = 144\pi$

2. The lateral area of a cone is its surface area excluding the base. If the lateral area of a cone is 60π square inches and the radius, height, and slant height are all integer values of inches, what are all the possible values for the volume of the cone?

Lateral area = $= \pi r l = \pi r(\sqrt{h^2 + r^2})$

$$60 = r\sqrt{h^2 + r^2}$$

The factors of 60 which are the third part of a Pythagorean triple are 5 and 10.

If $\sqrt{h^2 + r^2} = 5$, then r would have to be 12, which can't be the case.

If $\sqrt{h^2 + r^2} = 10$, then r could be 6 and h could be 8 in order to get a lateral area of 60.

Then the volume is $\frac{1}{3}\pi h r^2 = \frac{1}{3}\pi(8)(6^2) = 96\pi$

3. A cubical box of side 1.5 feet is used to stack cylindrical cans in layers of 6 cans x 6 cans. Each can has base radius 1.5 inches and height 4.5 inches. What is the volume of the space between the cube and the cylinders? Give your answer in cubic inches.

1.5 feet = 18 inches, so the volume of the cube is $18^3 = 5832$.
Since 18 divided by 4.5 is 4, you can stack 4 rows of cans in a 6x6 arrangement.
The volume of each can is $\pi(4.5)(1.5)^2 = 10.125\pi$,
and you have 6x6x4 = 144 cans, so $144 * (10.125\pi) = 1458\pi$
The space between is $5832 - 1458\pi$ cubic inches.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 6 Round 4
Radical Expressions and Equations

1.) $\pm\sqrt{19}$

2.) $\dfrac{187\sqrt[3]{4}}{4}$

3.) 6558, -2

1) Solve for all real values of x:

$$\sqrt[4]{x^2-3}+5=7$$

$$\sqrt[4]{x^2-3}=2$$

$$x^2-3=16$$

$$x^2=19$$

$$x=\pm\sqrt{19}$$

2) Express as a single fraction in simplest radical form:

$$4\sqrt[3]{32}-\frac{5}{\sqrt[3]{16}}+10\sqrt[3]{256}$$

$$=4(\sqrt[3]{8}\sqrt[3]{4})-\frac{5\sqrt[3]{4}}{\left(\sqrt[3]{16}\sqrt[3]{4}\right)}+10(\sqrt[3]{64}\sqrt[3]{4})$$

$$=8\sqrt[3]{4}-\frac{5\sqrt[3]{4}}{4}+40\sqrt[3]{4}$$

$$=\frac{32\sqrt[3]{4}-5\sqrt[3]{4}+160\sqrt[3]{4}}{4}$$

$$=\frac{187\sqrt[3]{4}}{4}$$

3) Solve for all real values of x: $4\sqrt[8]{x+3}-\sqrt[4]{x+3}=3$

This is $4(x+3)^{\frac{1}{8}}-(x+3)^{\frac{2}{8}}=3$. Let y = $(x+3)^{\frac{1}{8}}$. Then $4y - y^2 = 3$

So $y^2 - 4y + 3 = 0$, so $(y-3)(y-1) = 0$; Y=3 or y=1. If $(x+3)^{\frac{1}{8}}=3$, then $x+3=3^8=$ 6561, so so x=6558. If y=1, then $(x+3)^{\frac{1}{8}}=1$, and x+3=1, so x=-2

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 6 Round 5 Polynomials and Advanced Factoring

1.) ________2, 3, -5_______________

2.) ______$(x-2)(x+3)^2(x-3)^2$ _______

3.) __$(x^2 -2x + 5)(x^2 -3x + 9)$

1.) Find the 3 integer zeros of $x^3 - 19x + 30$
By the integer zero theorem, the only possibilities for integer zeros are ±1,2,3,5,6,10,15,30. x=2 clearly works since $2^3 - 19*2+30=0$. Divide $x^3-19x+30$ by x-2 to get $x^2 + 2x-15$, which factors to (x+5)(x-3), so the other two zeros are 3 and -5.

2.) Factor $x^5 - 2x^4 -18x^3 + 36x^2 +81x - 162$ into 5 linear binomials with integer coefficients.
Factor by grouping $x^4(x-2) - 18x^2(x-2) + 81(x-2)$
$=(x-2)(x^4-18x^2+81)$
$=(x-2)(x^2-9)^2$
$=(x-2)(x+3)(x+3)(x-3)(x-3)=(x-2)(x+3)^2(x-3)^2$

3) Given that 1+2i is a complex zero of $x^4 - 5x^3 + 20x^2 - 33x + 45$, factor $x^4 - 5x^3 + 20x^2 - 33x + 45$ into two quadratic trinomials with integer coefficients.

Since the fourth degree polynomial has integer coefficients, 1-2i must also be a zero. The quadratic polynomial with zeros 1+2i and 1-2i is $x^2 -2x + 5$. Divide the fourth degree polynomial by $x^2 - 2x + 5$ to get $x^2 -3x + 9$, so the factoring is $(x^2 -2x + 5)(x^2 -3x + 9)$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-16

Match 6 Round 6
Counting and Probability

1.) (4,2)

2.) $\frac{15}{23}$

3.) $\frac{17}{35}$

1) For what ordered pairs (N,R) is the number of permutations of N objects equal to four times the number of combinations of N objects taken R at a time? Express your answers as ordered pairs (N, R).

We need $n! = 4\frac{n!}{r!(n-r)!}$, so $r!(n-r)! = 4$

Neither (n-r)! nor r! can be 1, since that means the other one is 4, and there is no number M for which M!=4. r! can be 2 and (n-r)! can be 2, which means r=2 and n=4. This is the only possible ordered pair.

2.)_At a certain company, there are 60% of the workers are female and 40% of the workers are male. 25% of the female workers are classified as managers, while 20% of the male workers are classified as managers What is the probability that if you choose a manager at random that the manager is female?

Say we choose a number of employees such as 100. That gives 60 females and 40 males. We have 15 female managers and 8 male managers. The probability that a manager is a female is $\frac{15}{23}$

3.) A regional board of education representing 2 towns has 4 members from each town. If a committee of 4 is randomly selected from the 8 board members, what is the probability that the committee contains at least 3 members from either of the towns?

This is the complement of having exactly 2 board members from one town and 2 board members from the other. Compute this probability and subtract it from 1. That probability must be

$$\frac{{}_4C_2 * {}_4C_2}{{}_8C_4} = \frac{36}{70} = \frac{18}{35}$$

$$1 - \frac{18}{35} = \frac{17}{35}$$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 6 Team Round

1.) _C=_______3A________4.) ________-7________

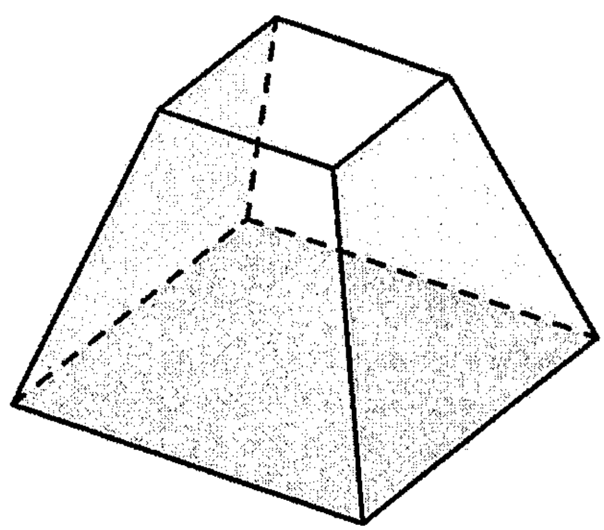
2.) ________90___cm^2 5.) $(5x^2+2y^2+10xy)(5x^2+2y^2-10xy)$

3.) _____$\frac{(\pi\sqrt{3}-2)L^3}{2}$____ 6.)_(5,3),(16,8) (OK to use the word "and")

1) Four times the measure of $\angle A$ is equal to the measure of the supplement of $\angle B$. The measure of $\angle B$ is equal to the sum of the measures of the complements of $\angle A$ and $\angle C$. Solve for the measure of $\angle C$ in terms of the measure of $\angle A$ if all measurements are in degrees. Use C for the measure of $\angle C$ and A for the measure of $\angle A$.

4A = (180-B) and B=(90-A)+(90-C), so B = 180-A-C. .
4A = (180-(180-A-C), so 4A=180-180+A+C. C=3A.

2). When the top of a pyramid is cut from another pyramid, the remaining solid is called a frustum. A square pyramid has the length of its base 6 cm and its height 4 cm. A slice is made parallel to the base of the pyramid to create a new pyramid which is one-eighth of the volume of the original pyramid. This pyramid is removed to create a frustum . Find the total



surface area of the frustum.

The original pyramid has volume $\frac{1}{3}*4*6^2=48$. If you cut off the top of the pyramid with height h, the ratio of its height to its base of the ratio of height to base in the original pyramid, so the ratio of height to base is $\frac{2}{3}$. $h=\frac{2}{3}b$. , where b is the top base of the frustum. The volume of the pyramid that was cut off must be 6, so $\frac{1}{3}(\frac{2}{3}b)b^2=6, b^3=27, b=3,\ h=2\ h=2$.

The slant height of the old pyramid was $\sqrt{3^2+4^2}=5$. Since the pyramid was cut off at the halfway between the top and the base, the slant height of the frustum will be 2.5.

The surface area consists of the area of the 2 bases plus the area of the 4 trapezoids.

Area = $3^2+6^2+4*(\frac{1}{2}(3+6)*2.5)=$

$9+36+45=90$

3) A cube of side length L is inscribed in a sphere. Find the volume between the sphere and the cube in terms of L. Give your answer as single fraction in simplest radical form.

The longest diagonal of a cube will be $L\sqrt{3}$, so the radius of the sphere will be $\frac{L\sqrt{3}}{2}$. The volume of the sphere will be $\frac{4}{3}\pi(\frac{L\sqrt{3}}{2})^3$ and the volume of the cube will be L^3.

$$\frac{4}{3}\pi(\frac{L\sqrt{3}}{2})^3-L^3=$$

$$\frac{4}{3}\pi\frac{L^3 3\sqrt{3}}{8}-L^3=\frac{\pi L^3\sqrt{3}}{2}-L^3=\frac{(\pi\sqrt{3}-2)L^3}{2}$$

4) Solve for all real numbers x such that $\sqrt{2-x}-\sqrt{x+11}=\sqrt{2x+15}$.

Square both sides:

$$(\sqrt{2-x}-\sqrt{x+11})^2=(\sqrt{2x+15})^2$$

$$2-x+x+11-2\sqrt{2-x}\sqrt{x+11}=2x+15$$

$$-2\sqrt{2-x}\sqrt{x+11}=2x+2$$

$$-\sqrt{2-x}\sqrt{x+11}=x+1$$

$$Square\_both\_sides\_again$$

$$(2-x)(x+11)=x^2+2x+1$$

$$-x^2-9x+22=x^2+2x+1$$

$$2x^2+11x-21=0$$

$$(x+7)(2x-3)=0$$

$$x=-7\_or\_x=\frac{3}{2}$$

$$\sqrt{0.5}-\sqrt{12.5}=\sqrt{18}$$

is clearly not true, so $x=\frac{3}{2}$ is extraneous, but x=-7 checks. 3-2=1.

5) Factor into 2 quadratic trinomials in x and y: $25x^4-80x^2y^2+4y^4$

$25x^4 - 80x^2y^2 + 4y^4 = 25x^4 + 20x^2y^2 + 4y^4 - 100x^2y^2 = (5x^2 + 2y^2)^2 - (10xy)^2 =$
$(5x^2 + 2y^2 + 10xy)(5x^2 + 2y^2 - 10xy)$

6.) A set of N marbles contains exactly B blue marbles. The probability of drawing exactly 3 marbles from the set of N marbles which are all blue is 0.1. If N≤20, find the two possible combinations of N and B for which this is true. Express your answers as ordered pairs (N,B).

We have $$\frac{\frac{B(B-1)(B-2)}{6}}{\frac{N(N-1)(N-2)}{6}} = 0.1, so$$

$$B(B-1)(B-2) = 0.1N(N-1)(N-2)$$

A simple solution is N=5, B=3, since 3*2*1= 0.1*5*4*3 =6.
Continue computing 3*2*1, 4*3*2, 5*4*3, … until you find the trickier one N=16, B=8.

$$\frac{8*7*6}{16*15*14} = \frac{4*2*7*3*2}{4*2*2*3*5*7*2} = \frac{1}{2*5} = 0.1$$