FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 6 Round 1
Geometry: Lines and Angles

Note: Figures not necessarily
Drawn to scale

1.) $m(\angle J) = 19°, m(\angle K) = 71°$

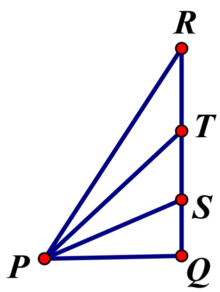
2.) ______________240______________

3.) ___________140______________

1.)_Angle J is complementary to angle K. Twice the degree measure of the supplement of angle J is 5 less than 3 times the degree measure of the supplement of angle K. Find the degree measures of angle J and angle K.

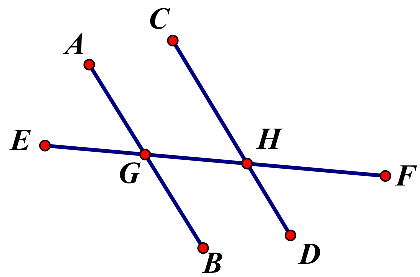
Let J = degree measure of angle J, K = degree measure of angle K
2(180-J) = 3(180-K)-5, but J=90-K, so
2(180-(90-K))=3(180-K)-5
2(90+K)=540-3K-5
180+2K=535-3K
5K = 355, so K = 71 degrees. J = 19 degrees.

2.)_ In right triangle PQR, the right angle is at Q. $\overrightarrow{PT}$ and $\overrightarrow{PS}$ serve to trisect $\angle RPQ$. If the measure of $\angle RTP$ is 10 less than 7 times the measure of $\angle SPQ$, find the sum of the measures of $\angle TSP$ and $\angle RTP$ in degrees.



Let the degree measure of angle SPQ be x. Then so is the measure of angle TPS and the measure of angle RPT. Then angle PRQ (same as angle PRT) is 90-3x degrees. Angle RTP is 180-x-(90-3x) = 90+2x. If 90+2x=7x-10, then 5x=100, and x=20. m($\angle SPQ$)=20 degrees, and angle RTP has measure 90+2*20 = 130 degrees. Now m($\angle TSP$) is 180-x-(180-(90+2x))= 180-x-(90-2x)=90+x. So angle TSP has measure 110 degrees. The sum of the two angles is 240 degrees.

3.) $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$. The lines are cut by transversal $\overleftrightarrow{EF}$, which intersects line $\overleftrightarrow{AB}$ at G and $\overleftrightarrow{CD}$ at H. If $m\angle AGE$ = (5x-10) degrees and $m\angle CHF$ = (x^2+40) degrees, find all possible values of $m\angle CHF$ in degrees.



Angles AGE and CHF must be supplementary, since angle AGE is supplementary to angle AGF, and angles AGF and CHF are corresponding angles for two parallel lines cut by a transversal. So (5x-10) + (x^2 +40) = 180.

$X^2 + 5x + 30 = 180$

$X^2 + 5x - 150 = 0$

(x+15)(x-10)=0, so x=-15 or x=10. The -15 solution doesn't work for angle AGE, so x=10. Angle CHF is 10^2+40 = 140 degrees.

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Match 6 Round 2
Algebra: Literal Equations

1.) ___ $y = \frac{-3}{5}x + \frac{13}{5}$

2.) _ $x = \frac{-5 \pm \sqrt{25 - 4f}}{2}$

3.) ____b=a+4__________________

1.)_ If x=-5t+1 and y=3t+2 represent the parametric equations of a line, express y in terms of x. Give your answer in y=mx+b form.

X=-5t+1, so $t = \frac{x-1}{-5}$, so $y = 3\left(\frac{x-1}{-5}\right) + 2$, so $y = \frac{-3}{5}x + \frac{13}{5}$

2.)_ If no denominators are equal to zero and f<0 , solve for x in terms of f:

$$\frac{1}{x+3} = \frac{1}{f-6} + \frac{1}{x+2}$$

Multiply through by (x+3)(f-6)(x+2) to get
(x+2)(f-6) = (x+2)(x+3) + (x+3)(f-6).
The x(f-6)'s cancel and you are left with (f-6) on the right, so
0 = (x+2)(x+3) + (f-6)
$0 = X^2 + 5X + 6 + f - 6$
$0 = x^2 + 5x + f$

From quadratic formula, $x = \frac{-5 \pm \sqrt{25 - 4f}}{2}$ Since f<0, we know the expression under the square root is positive.

3.) If a≠2 and b≠-2, solve $(a-2)b^2 - 4a = a^2b - 4b + 2(a^2 - 8)$ for b in terms of a.
This is $(a-2)b^2 - 4a = a^2b - 4b + 2a^2 - 16$. Rewrite this as
$(a-2)b^2 - 4a + 8 = a^2b - 4b + 2a^2 - 8$, which factors by grouping to
$(a-2)b^2 - 4(a-2) = b(a^2-4) + 2(a^2-4)$, so
$(a-2)(b^2-4) = (b+2)(a^2-4)$, so (a-2)(b+2)(b-2) = (b+2)(a-2)(a+2). We can cancel the a-2 and b+2 since a≠2 and b≠-2, so b-2 = a+2, and b=a+4

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Match 6 Round 3
Geometry:
Solids and
Volumes

1.) ________36 ____________

2.) ________13824-2304π ____

3.) ________ $\frac{4\sqrt{6}}{3}$ __________

1) A cone of radius 2 cm and height 3 cm is used to scoop water into an empty cylinder of radius 6 cm and height 4 cm. How many full scoops of water from the cone will it take to fill up the cylinder?

The cone has volume $\frac{1}{3}p(2^2)(3) = 4p$. The cylinder has volume $p(6^2)(4) = 144p$

$\frac{144p}{4p} = 36$

2. A cubical box with sides 2 feet long is used to pack 27 spherical cannonballs that each have diameter 8 inches. How many cubic inches of the volume of the box will not be taken up by the cannonballs? Express your answer in terms of π.

The radius of each cannonball is 4 inches, so the volume of each cannonball is $\frac{4}{3}p(4^3) = \frac{256p}{3}$

Since there are 27 of them, the total volume is $27 * \frac{256p}{3} = 2304p$. The side of the box is 24 inches, so the total volume of the box is 24^3 = 13824.

So the total volume not taken up by the cannonballs is 13824-2304π.

3) The line segment from (0,0) to (4,4) is rotated around the y-axis to create a cone. The plane y=k intersects the cone such that the lateral area of the part of the cone from y=0 to y=k is twice the lateral area of part of the cone from y=k to y=4. What is the value of k?

The original cone has height 4 and radius 4, so the slant height is $\sqrt{4^2 + 4^2} = 4\sqrt{2}$, and the lateral area is $\pi*4*4\sqrt{2} = 16\pi\sqrt{2}$. The part of the cone below y=k, has height k and radius k, so its slant height is $k\sqrt{2}$, and its lateral is $\pi*k*k\,k\sqrt{2} = \pi k^2\sqrt{2}$.

We need $\pi k^2\sqrt{2} = \frac{2}{3}*16\pi\sqrt{2}$, so $k^2 = \frac{32}{3}$, and $k = \sqrt{\frac{32}{3}} = \frac{4\sqrt{2}}{\sqrt{3}} = \frac{4\sqrt{6}}{3}$

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Match 6 Round 4
Radical Expressions and Equations

1.) ___________ $\frac{737\sqrt{3}}{36}$ _________

2.) ________________ $2\sqrt[6]{54}$ _____________

3.) _________________ 66 ___________

1) Simplify as much as possible: $4\sqrt{75}+\frac{8}{\sqrt{27}}-5\sqrt{\frac{1}{48}}$

$4\sqrt{75}+\frac{8}{\sqrt{27}}-5\sqrt{\frac{1}{48}} = 20\sqrt{3}+\frac{8}{3\sqrt{3}}-\frac{5}{4\sqrt{3}}$ $20\sqrt{3}+\frac{8\sqrt{3}}{9}-\frac{5\sqrt{3}}{12}=$

$\frac{720\sqrt{3}+32\sqrt{3}-15\sqrt{3}}{36}=\frac{737\sqrt{3}}{36}$

2) Express the following as the product of an integer greater than 1 and the sixth root of an integer: $\sqrt[3]{4}\sqrt{6}$

$$\sqrt[3]{4}\sqrt{6}=4^{\frac{1}{3}}*6^{\frac{1}{2}}=2^{\frac{2}{3}}*6^{\frac{1}{2}}=$$

$$2^{\frac{4}{6}}*6^{\frac{3}{6}}=2^{\frac{4}{6}}*2^{\frac{3}{6}}*3^{\frac{3}{6}}=$$

$$2*2^{\frac{1}{6}}*3^{\frac{3}{6}}=2\sqrt[6]{54}$$

3) Solve for all real values of x: $\sqrt[3]{x-2}-\sqrt[6]{x-2}=2$

This is $(x-2)^{\frac{2}{6}}-(x-2)^{\frac{1}{6}}=2$. Let y = $(x-2)^{\frac{1}{6}}$. Then $y^2 - y - 2 = 0$

So (y-2)(y+1)=0. Y=2 or y=-1. If $(x-2)^{\frac{1}{6}}$=2, then x-2=64, and x=66.

If $(x-2)^{\frac{1}{6}}$=-1, alarm bells should start going off, but if not, raise both sides to the sixth power and x-2=-1, so x=1. Substituting x=1 gives the sixth root of a negative number, so the only answer is x=66.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 6 Round 5
Polynomials and Advanced Factoring

1.) ________________-2, 4, 5_______

2.) ______b= -11___ c= 40_______

3.) ____________0____________

1.) Find all real zeros of $x^3 – 7x^2 + 2x + 40$.
Try x=-2, and you get a zero by synthetic division, with the other polynomial as $x^2-9x+20$, which factors to (x-4)(x-5), so the three zeros are -2, 4, 5

2.) A cubic polynomial in x with integer coefficients and leading coefficient 1 has one of its complex zeros equal to 3+i, and the constant term of the polynomial is -50.
If the polynomial is expanded as $x^3 + bx^2 + cx – 50$, what are b and c?
Since the polynomial has integer coefficients, if 3+i is a zero, so is 3-i, so a quadratic with these zeros is $x^2-6x+10$.
Since the constant is -50, the other factor must be x-5
Multiplying these gives $x^3 -11x^2 + 40x -50$. b=-11, c=40

3) Give the sum of the six complex zeros of $a^6 – 7a^3 - 8$.
This factors to $(a^3-8)(a^3+1)=(a-2)(a^2+2a-4)(a+1)(a^2-a+1)$. Solving $a^2+2a+4 = 0$ gives a = $\frac{-2\pm\sqrt{-12}}{2} = -1\pm i\sqrt{3}$, and solving $a^2-a+1=0$ gives $\frac{-1\pm i\sqrt{3}}{2}$.
Adding $2 + -1+i\sqrt{3} + -1-i\sqrt{3} + -1 + \frac{1-i\sqrt{3}}{2} + \frac{1+i\sqrt{3}}{2} = 0$, which makes sense if you think about the sum of the cube roots of any number vector-wise using DeMoivre’s Theorem

FAIRFIELD COUNTY MATH LEAGUE (FCML)

Match 6 Round 6
Counting and Probability

2014-2015

1.) ___________25__________

2.) _________ $\frac{5}{9}$ ___________

3.) ___________ $\frac{31}{42}$ ____________

1) What is the smallest value of n such that n! is evenly divisible by one million?
You need at least 6 factors of 2 and 6 factors of 5 in n!. It is simple enough to get factors of 2 – you get those every time n is even. You don't get 6 factors of 5 until you get to 25! – then there is one factor of 5 in 5, 10, 15, and 20, and then 2 factors of 5 in 25, so 25! Will be the smallest n such that n! is evenly divisible by one million.

2) A couple has two children. If you know that at least one child is a boy and at least one child was born after 12:00 noon, what is the probability that both children are boys or both children were born after 12:00 noon?
There are 2x2x2x2 = 16 equally likely possibilities. For example, GBAP would be the event first child is a girl and second child is a boy and the girl was born in the before noon (AM) and the boy was born after noon (PM). There are 9 outcomes where you have the one child a boy and at least one child was born after 12:00 noon: BBPP, GBAP, BGAP, GBPA, BGPA, BBAP, BBPA, GBPP, and BGPP. The 5 outcomes which meet the given criterion are BBAP, BBPA, BGPP, GBPP, and BBPP. So the answer is $\frac{5}{9}$.

3) A regional commission has 5 members from Town A, 3 members from Town B, and 2 members from Town C. If they randomly choose a committee of 4 members, what is the probability that there are at least 2 members from Town A on the committee?
There are 10C4 = (10*9*8*7)/(4*3*2*1)=210 possible committees. The probability of getting exactly 2 members from Town A is $\frac{C(5,2)*C(5,2)}{C(10,4)} = \frac{100}{210}$. The probability of getting exactly 3 members form Town A is $\frac{C(5,3)*C(5,1)}{C(10,4)} = \frac{50}{210}$.

The probability of getting exactly 4 members from town A is $\frac{C(5,4)*C(5,0)}{C(10,4)} = \frac{5}{210}$

The total probability is $\frac{155}{210} = \frac{31}{42}$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2014-2015

Match 6 Team Round

1.) _C=_______2A__________ 4.) _$(a+5)(a-1)(x-2)(x-1)$

2.) ___________ $\frac{7}{12}\pi$ ______ 5.) ___________ $\frac{2}{5}$ _____________

3.) ___________3__________ 6.) ______N=4_____B=3_______

1) Four times the measure of $\angle A$ is equal to the sum of the measures of the supplements of $\angle B$ and $\angle C$. Half the measure of $\angle B$ is equal to the sum of the measures of the complements of $\angle A$ and $\angle C$. Solve for the measure of $\angle C$ in terms of the measure of the $\angle A$ if all measurements are in degrees. Use C for the measure of $\angle C$ and A for the measure of $\angle A$.

   4A = (180-B)+(180-C) and $\frac{B}{2}$=(90-A)+(90-C), so B = (180-2A)+(180-2C) = 360-2A-2C. 4A = (180-B)+(180-C) simplifies to 4A = 360-B-C. Substitute this expression for B into the previous equation to get 4A = 360 – (360-2A-2C) – C, so 4A = 2A+2C-C, so C=2A.

2) A cylinder with height R is inscribed in a sphere of radius R. The volume that is inside the sphere but outside the cylinder is some constant multiplied by R^3. Give the value of that constant.

   The center of the cylinder must be in the center of the sphere for it to be an inscribed cylinder, so the distance from the center of cylinder to the top or bottom of the cylinder is $\frac{R}{2}$. Draw a line segment from the center of the cylinder to the point where the cylinder meets the sphere. This must have length R, so to find the radius r of the cylinder, use Pythagorean theorem with $r^2+(\frac{R}{2})^2=R^2$, so r = $\frac{R\sqrt{3}}{2}$. The volume of the cylinder will then be $R(\pi)(\frac{R\sqrt{3}}{2})^2=\frac{3\pi R^3}{4}$. The volume of the sphere is $\frac{4\pi R^3}{3}$, so the constant is $(\frac{4}{3}-\frac{3}{4})\pi=\frac{7}{12}\pi$

3) Solve for all real numbers x such that $\sqrt{x+6}-\sqrt{x+1}=\sqrt{x-2}$.

$$a^2x(x-3)+2a(a+2x^2)-4a(3x-2)-5(x^2+2)+15x$$

Square both sides: $a^2x^2-3a^2x+2a^2+4ax^2-12ax+8a-5x^2-10-15x$

$$a^2(x^2-3x+2)+4a(x^2-3x+2)-5(x^2$$

$$(\sqrt{x+6}-\sqrt{x+1})^2=(\sqrt{x-2})^2$$

$$x+6+x+1-2\sqrt{x+6}\sqrt{x+1}=x-2$$

$$-2\sqrt{x+6}\sqrt{x+1}=-x-9$$

$$Square\_both\_sides\_again$$

$$4(x+6)(x+1)=x^2+18x+81$$

$$4x^2+28x+24=x^2+18x+81$$

$$3x^2+10x-57=0$$

$$(x-3)(3x+19)=0$$

$$x=3\_or\_x=\frac{-19}{3}$$

$$\frac{-19}{3}\_is\_extraneous\_so\_x=3$$

4) Factor into 4 first degree polynomials with integer coefficients:

$$a^2x(x-3)+2a(a+2x^2)-4a(3x-2)-5(x^2+2)+15x$$

$$a^2x(x-3)+2a(a+2x^2)-4a(3x-2)-5(x^2+2)+15x$$

$$a^2x^2-3a^2x+2a^2+4ax^2-12ax+8a-5x^2-10+15x$$

$$a^2(x^2-3x+2)+4a(x^2-3x+2)-5(x^2-3x+2)=$$

$$(a^2+4a-5)(x^2-3x+2)=$$

$$(a+5)(a-1)(x-2)(x-1)$$

5.) The letters of the word ANAGRAM are jumbled and rearranged randomly. If you know that the first letter of the rearrangement is A, what is the probability that the rearrangement contains no A's that are adjacent to each other?

We could treat the A's as indistinguishable from each other or as A1, A2, and A3, and we would get the same answer because in the ratio the extra arrangements would cancel out. If C stands for consonant, the only possible arrangements where two A's are not adjacent are ACACACC, ACCACAC, ACACCAC, ACACCCA, ACCACCA, ACCCACA. Overall there are $C(6,2) = 15$ different ways to create such an arrangement, because you choose 2 of the 6 remaining places to place the A's, and the order in which you choose which place is not important, and since we're looking for probability, the different orders will cancel out in the numerator and denominator, and we get $\frac{6}{15} = \frac{2}{5}$. If we want to check each possibility, for each of the six, there are 3 choices of which A to use for the first letter, then 2 choices for the second A, and 1 choice for the third A. There are 4! Ways to use the 4 consonants. So multiply 6*3*2*4*3*2*1 = 864
Overall there are $C(6,2) = 15$ different ways to create such an arrangement, because you choose 2 of the 6 remaining places to place the A's, and the order in which you choose which place is not important. For each of the 15 arrangements, you can place the 3 A's in 3! Different orders, 4 consonants in 4! Different orders, so multiply 6*24*15 = 2160, so the answer is $\frac{864}{2160} = \frac{2}{5}$

6.) A set of N marbles contains exactly B blue marbles. The probability of drawing exactly 2 blue marbles from the set of N marbles is 0.5. If N≤20, what are the values of N and B?

When you choose two items out of a group of K elements, the number of ways it can be done is equivalent to the (K-1)st triangular number for K>2, since the formula is $\frac{K(K-1)}{2}$. The number of ways of choosing 2 blue marbles out of the B blue marbles is $\frac{B(B-1)}{2}$ and the number of ways of choosing any two marbles out of the N is $\frac{N(N-1)}{2}$. The ratio of these numbers is the probability, and that must equal 0.5. Start writing out the triangular numbers 1,3,6,10,15,21,… and the only combination of two triangular numbers such that one is half of the other is 3 and 6, which corresponds to the number of ways of choosing 2 items out of 3, and the number of ways of choosing 2 items out of 4, so B=3 and N=4. You don't reach another such combination until N=21 and B=15, so since N≤20, the only solution is N=4, B=3.