FAIRFIELD COUNTY MATH LEAGUE 2018-2019

Match 4 Round 1 Arithmetic: Basic Statistics

1.) 9

2.) 5

3.) $29

1.) What is the positive difference between the arithmetic mean and the median of the set of numbers of the form 2^N where N is an integer and $1 \leq N \leq 6$?

2.) The geometric mean of a set of numbers $\{a_1, a_2 ..., a_N\}$ is $\sqrt[N]{a_1 a_2 ... a_N}$. The geometric mean of 6 numbers is 45. Five of the numbers are 9, 27, 125, 225 and 243. What is the sixth number?

3.)_ You have a cart containing eight identical items that cost \$N each and two identical items that cost \$(3N+2) each. If you remove one of each type of item from the cart, the arithmetic mean of the remaining eight items is \$1.50 less than the arithmetic mean of the original ten items. Find the cost of the highest priced item.

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Match 4 Round 2
Algebra 1:
Quadratic
Equations

1.) $\frac{3 \pm \sqrt{14}}{2}$

2.) $\pm 10, \pm 17$

3.) $2, -\frac{1}{2}$

1.) Solve for x: $4x(x-3)=5$.

2.)_ For which integer values of k does the equation $4x^2 + kx + 4 = 0$ have two distinct rational solutions?

3.) Find all values of k such that the equation $1.5 - kx(6+(k+2)x) = 3k$ has exactly one real solution for x.

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Match 4 Round 3
Geometry:
Similarity

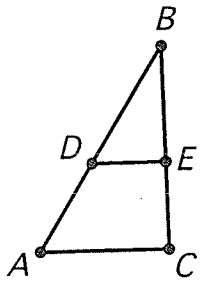
Note: Diagrams are not
Necessarily drawn to scale

1.) $\frac{20}{3}$

2.) $\frac{200\sqrt{11}}{11}$ cm

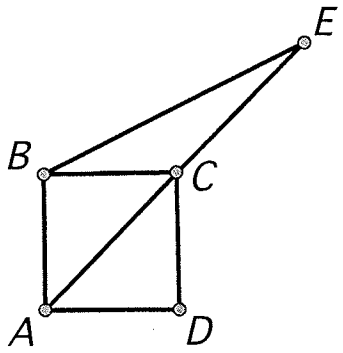
3.) $\sqrt{5}$ cm

1. ΔABC is a right triangle with the right angle at C. D and E are on $\overline{AB}$ and $\overline{BC}$ respectively. $\overline{DE}$ is parallel to $\overline{AC}$. AC=6, BC=8, and DE= 4. Find BD.



2.)_ The ratio of the areas of two regular decagons is 16:11. One side of the smaller decagon measures 5 cm. Find the perimeter of the larger decagon.

3.)_ The area of square ABCD is 1 cm^2. Diagonal $\overline{AC}$ is extended to E such that AC = CE. B and E are connected by a line segment. Find BE.



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Match 4 Round 4
Algebra 2:
Variation

1.) $-\frac{2}{5}$

2.) $\pm\frac{3\sqrt{35}}{7}$

3.) 900,000

1.) (z+4) varies inversely with the cube root of w. If z=2 when w=27, what is the value of z when w=125?

2.) m varies directly with n^2, and n varies inversely with p. If p=3 when m=5, what is p when m=7 ?

3.) The average number of telephone calls per day between two cities varies jointly with the populations of the two cities and inversely with the square of the distance between them. New York has population 8.5 million and Philadelphia has population 1.6 million. They are 100 miles away from each other. The average number of calls per day between New York and Philadelphia is 27.2 million. The distance between New York and Chicago is 700 miles. The population of Chicago is 2.7 million. What is the average number of phone calls per day between New York and Chicago rounded to the nearest hundred thousand?

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Match 4 Round 5
Trig Expressions
and DeMoivre's
Theorem

1.) $\frac{3\sqrt{3}-3}{4}$

2.) $2\sqrt{2}\,\text{cis}\left(\frac{5\pi}{12}\right)$

3.) 128

1.) Evaluate $\dfrac{\sin(\frac{\pi}{2})+\cos(\frac{\pi}{3})}{\tan(\frac{\pi}{4})+\cot(\frac{\pi}{6})}$,

2.) Find the cube root of -16-16i that is in the first quadrant. Express your answer as r cis(ø) where r is in simplest radical form and ø is in radians.

3.) If $(\cos^4(2\text{ø}))(\sin(2\text{ø}))^2$ is expressed using $\cos(\text{ø})$ as the only trig function, what is the coefficient of $\cos^6(\text{ø})$?

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Match 4 Round 6 Conics

1.) _.) Center: $\left(-\frac{1}{3}, 1\right)$ Radius: $\frac{\sqrt{10}}{3}$

2.) $\left(-\frac{15}{8}, 4\right)$

3.) $-\frac{1}{4}$

1.) Find the center and radius of the circle
$9x^2+6x+9y^2-18y=100$

2.) A parabola of the form $x=ay^2 + by+c$ has vertex at (-2,4) and passes through (6,6). Find the focus of the parabola.

3.) A hyperbola has foci at $(2\sqrt{5},0)$ and $(-2\sqrt{5},0)$ and passes through the point $(4\sqrt{2},2)$. Find the product of the slopes of the asymptotes of the hyperbola.

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1.) 52.6

4.) k= $-\frac{1}{4}$ n= $\frac{4}{3}$

2.) $4x^2+2x-1=0$

5.) $\frac{-28}{85}$

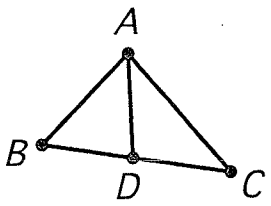
3.) 2

6.) $\left(\frac{1+\sqrt{7}}{4}, \frac{-1+\sqrt{7}}{2}\right)$

1.) The inter-quartile range of a set of 5 distinct numbers is the second highest number minus the second lowest number. Give the mean of the set of five consecutive prime numbers that has the largest inter-quartile range if all the prime numbers must be less than 100.

2) One quadratic equation has solutions $1\pm\sqrt{5}$. A second quadratic equation has solutions that are the reciprocals of the solutions of the first equation. Give the second equation in the form $y=ax^2+bx+c=0$ where a,b,c are relatively prime integers and a>0.

3.) In the diagram of ΔABC below, not necessarily drawn to scale, $\overline{AD}$ bisects $\angle BAC$. AC=3x+2, DC=x+4, BC=3x+9, AB=7x-2. Give all possible values of x.



4.) A direct power variation function of the form $y=kx^n$ passes through the points (64,-64) and (27,-20.25). Find the values of k and n.

5. A and B are first quadrant angles with $\tan(A)=3$ and $\tan(B)=4$. Find $\cos(2A)+\sin(2B)$.

6.) An ellipse has foci at $(0,\sqrt{3})$ and $(0,-\sqrt{3})$ and y-intercepts at $(0,2)$ and $(0,-2)$. One of the intersection points of the ellipse and the line $y=2x-1$ is in the first quadrant. Give the coordinates (x,y) of this point.