FAIRFIELD COUNTY MATH LEAGUE 2016-2017

Match 5 Round 1
Algebra I:
Fractions and
Exponents

1.) _______1.414_____

2.) ________ $\frac{25}{7}$ ___________

3.) ______ $\frac{-236a^4b^8c^{12}}{27}$ ______

1) Express as a decimal rounded correctly to three decimal places:

$$1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}}$$

$$1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}}=$$

$$1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{\frac{5}{2}}}}=$$

$$1+\cfrac{1}{2+\cfrac{1}{2+\frac{2}{5}}}$$

$$=1+\cfrac{1}{2+\cfrac{1}{\frac{12}{5}}}=1+\cfrac{1}{2+\frac{5}{12}}=1+\cfrac{1}{\frac{29}{12}}=$$

$$1+\frac{12}{29}=1.414$$

This is the beginning of the continued fraction expansion of $\sqrt{2}$.

2) Express as an integer or reduced fraction: $\dfrac{(25)^{-5}(49)^{-3}(35)^{4}}{(245)^{-2}(\frac{7}{25})^{3}}$

$$=\frac{(5^2)^{-5}(7^2)^{-3}(5*7)^4}{(5*7^2)^{-2}(5^{-2}*7)^3}=$$

$$b\,\frac{(5)^{-10}(7^{-6})(5)^4(7)^4}{(5^{-2}*7^{-4})(5^{-6}*7^3)}=$$

$$\frac{5^{-6}7^{-2}}{5^{-8}7^{-1}}=\frac{5^2}{7}=\frac{25}{7}$$

3.)_Express the following as a single fraction with no negative exponents:

$$\frac{(a)^3b^8(8c)^6}{27a^{-1}(\frac{1}{4}c)^{-6}}-\frac{(5a^2b^3c^4)^2(\frac{1}{3}c)^4}{(6b)^{-2}}$$

$$\frac{1}{27}a^4b^84^6c^6(\frac{1}{2})^6c^6-25a^4b^6c^8((\frac{1}{81})c^4)(36b^2)=$$

$$\frac{64a^4b^8c^{12}}{27}-\frac{900a^4b^8c^{12}}{81}=\frac{192a^4b^8c^{12}}{81}-\frac{900a^4b^6c^{12}}{81}=$$

$$\frac{-708a^4b^8c^{12}}{81}=\frac{-236a^4b^8c^{12}}{27}$$

=

FAIRFIELD COUNTY MATH LEAGUE 2016-2017

Match 5 Round 2
Algebra I:
Fractional
Expressions and
Equations

1.) ______ $\frac{3}{4}$ _________

2.) ___________ -5, 3 __________

3.) ___ $\frac{3}{2(x-2)(x-3)}$ ___

1). Simplify the product as much as possible if no values of x make any denominators equal to zero:

$$\frac{x^2+6x-27}{6x^2-16x-6} * \frac{9x^2-15x-6}{2x^2+14x-36}$$

$$\frac{(x+9)(x-3)}{2(3x+1)(x-3)} * \frac{3(3x+1)(x-2)}{2(x+9)(x-2)} = \frac{3}{4}$$

2). Solve for all possible values of x:

$$3 - \frac{4}{x+3} = \frac{2x+15}{x+6}$$

$$3-\frac{4}{x+3}=\frac{2x+15}{x+6}$$

$$3(x+3)(x+6)-4(x+6)=(2x+15)(x+3)$$

$$3x^2+27x+54-4x-24=2x^2+21x+45$$

$$x^2+2x-15=0$$

$$(x+5)(x-3)=0$$

$$x=-5, x=3$$

3.) Simplify as much as possible:

$$\frac{1}{2x^2-10x+12}-\frac{3}{2x^2-5x+2}-\frac{5}{-2x^2+7x-3}$$

=

$$\frac{1}{2(x-2)(x-3)}-\frac{3}{(2x-1)(x-2)}+\frac{5}{(2x-1)(x-3)}=$$

$$\frac{2x-1-6(x-3)+10(x-2)}{2(x-2)(x-3)(2x-1)}=\frac{6x-3}{2(x-2)(x-3)(2x-1)}=$$

$$\frac{3(2x-1)}{2(x-2)(x-3)(2x-1)}=\frac{3}{2(x-2)(x-3)}$$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2016-2017

Match 5 Round 3 Geometry: Circles

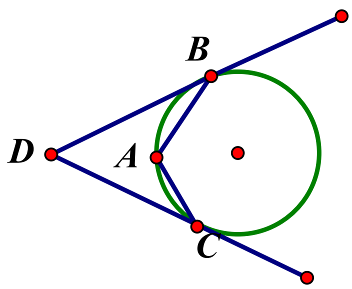
1.) ________20_______degrees

2.) __________8_____________

Note: Diagrams not necessarily to scale.

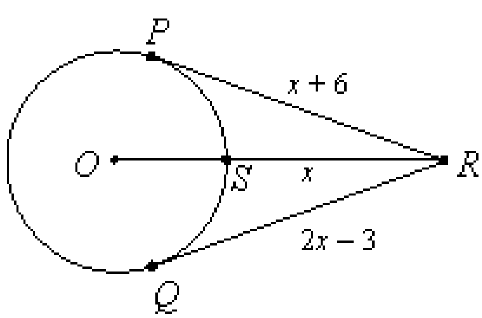
3.) _____ 100π___________cm^2

1.) In the picture below, ∠*BAC* is inscribed in the circle and $\overline{DB}$ and $\overline{DC}$ are tangent to the circle. If the measure of ∠*BAC* is 100 degrees, find the measure of ∠*BDC* in degrees.



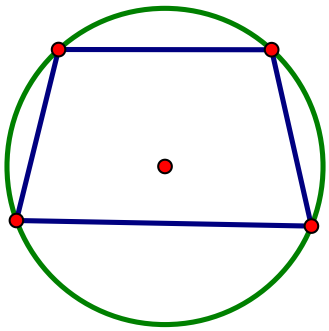
Since ĐBAC is an inscribed angle, the major arc opposite ĐBAC is 200 degrees, so the minor arc is 160 degrees. ĐBDC measures half the difference of the intercepted arcs, so its measure is $\frac{200 - 160}{2} = 20$ degrees.

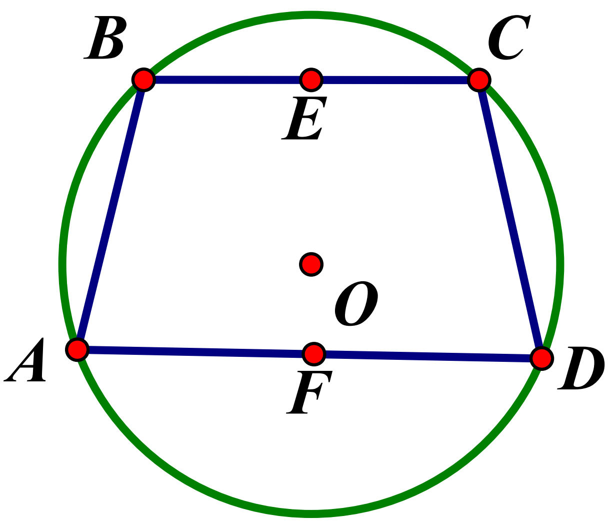
2.) The circle below has center O. $\overline{RP}$ and $\overline{RQ}$ are tangent to the circle. $\overline{RO}$ intersects the circle at S. If SR=x, PR=x+6, and RQ=2x-3, find the radius of the circle.



PR=RQ, so x+6=2x-3, so x=9. If the radius of the circle is r, then for ΔOPR, $r^2 + (9+6)^2 = (9+r)^2$ so $r^2 + 225 = r^2 + 18r+81$, 18r=144, r=8.

3. A circle is circumscribed about an isosceles trapezoid with base lengths 12 cm and 16 cm. The area of the trapezoid is 196 cm^2. Find the area of the circle.





Consider the vertical line $\overline{EF}$ through the center of the circle with circle O that has EF=height of trapezoid. Find the height EF by

$$196 = \frac{1}{2}(12+16)(EF)$$

$$196 = 14(EF)$$

$$EF = 14$$

Since E and F are at the midpoints of the bases, EC=6 and FD=8. Let OE=x and consider ΔOCE and ΔODF. OC and OD are radii, so let the radius of the circle be r. Then

$$x^2 + 6^2 = r^2$$

$$(14 - x)^2 + 8^2 = r^2$$

$$\_so\_x^2 + 36 = 196 - 28x + x^2 + 64$$

$$28x = 224$$

$$x = 8$$

so the radius of the circle is $\sqrt{8^2 + 6^2} = 10$, so the area is 100π cm^2.

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Match 5 Round 4
Quadratic Equations and Complex Numbers

1.) ______8-2i______________

2.) _____-14 < k < 2____

3.) _________ $i, \frac{1}{2}$ ________

1) Simplify: $\frac{5i^7 + (3i+4)(6i+7)}{5i}$

$$\frac{5i^7 + (3i+4)(6i+7)}{5i}$$

$$= \frac{-5i + (18i^2 + 24i + 21i + 28)}{5i}$$

$$= \frac{10+40i}{5i} = \frac{2+8i}{i} = \frac{i(2+8i)}{i^2} = \frac{2i-8}{-1} = 8 - 2i$$

2) For what values of k will the equation $x^2 + 3kx + 2k^2 = -2x - 8$ have two distinct complex solutions? You may answer in inequality notation or interval notation.

$$x^2 + 3kx + 2k^2 = -2x - 8$$

$$x^2 + (3k+2)x + (2k^2+8) = 0$$

We need the discriminant less than 0, so

$$x^2 + 3kx + 2k^2 = -2x - 8$$
$$x^2 + (3k + 2)x + (2k^2 + 8) = 0$$
$$(3k + 2)^2 - 4 * (2k^2 + 8) < 0$$
$$9k^2 + 12k + 4 - 8k^2 - 32 < 0$$
$$k^2 + 12k - 28 < 0$$
$$(k + 14)(k - 2) < 0$$
$$-14 < k < 2$$

3) Solve for all complex z: $2z^2 - (1 + 2i)z + i = 0$

$$z = \frac{(1 + 2i) \pm \sqrt{(1 + 2i)^2 - 4 * 2 * i}}{2 * 2}$$
$$= \frac{(1 + 2i) \pm \sqrt{-3 + 4i - 8i}}{4}$$
$$= \frac{(1 + 2i) \pm \sqrt{-3 - 4i}}{4}$$

To find $\sqrt{-3 - 4i}$, set $(a+bi)^2 = -3-4i$
$a^2 - b^2 = -3$
2ab=-4, so a=1, b=-2 or a=-1, b=2. Since we have the plus or minus anyway, that takes care of both possibilities.

$$= \frac{(1 + 2i) + (1 - 2i)}{4} \_or\_ \frac{(1 + 2i) - (1 - 2i)}{4}$$
$$= \frac{1}{2} \_or\_ i$$

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Match 5 Round 5
Solving Trig
Equations

1.) ______ $\frac{\pi}{10}, \frac{\pi}{2}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{17\pi}{10}$ __

2.) __________ $0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$ ________

3.) _________ $0, \frac{4}{5}$ _________

1) Solve for all x if $0 \le x < 2\pi$: $\sin(5x) = 1$

$$5x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \frac{17\pi}{2}$$

x= $\frac{\pi}{10}, \frac{\pi}{2}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{17\pi}{10}$

2) Solve for all x $0 \le x < 2\pi$ if $\cos(2x) - \sin(x) = 1$
‘

$$\cos(2x) - \sin(x) = 1$$

$$1 - 2\sin^2(x) - \sin(x) = 1$$

$$-2\sin^2(x) - \sin(x) = 0$$

$$-\sin(x)(2\sin x + 1) = 0$$

$$x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$$

3.) If cos(x)+2sin(x)=2, what are all possible values for cos(x)?

$$\cos(x) + 2\sin(x) = 2$$
$$\cos(x) - 2 = -2\sin(x)$$
$$\cos^2(x) - 4\cos(x) + 4 = 4\sin^2(x)$$
$$\cos^2(x) - 4\cos(x) + 4 = 4(1 - \cos^2(x))$$
$$5\cos^2(x) - 4\cos(x) = 0$$
$$\cos(x)(5\cos(x) - 4) = 0$$
$$\cos(x) = 0\_or\_\cos(x) = \frac{4}{5}$$

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Match 5 Round 6
Sequences and Series

1.) $\frac{5}{6}$

2.) $2\sqrt{3}$, $-2\sqrt{3}$

3.) $\frac{3}{5}, \frac{-1}{45}$

1.) Evaluate

$$\sum_{k=1}^{5} \frac{1}{k(k+1)}$$

$$\frac{1}{1*2}+\frac{1}{2*3}+\frac{1}{3*4}+\frac{1}{4*5}+\frac{1}{5*6}=$$

$$\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}$$

$$\frac{30}{60}+\frac{10}{60}+\frac{5}{60}+\frac{3}{60}+\frac{2}{60}=\frac{50}{60}=\frac{5}{6}$$

2.) The third term of a geometric sequence of real numbers is 162 and the seventh term is 18. What are the possible values for the tenth term of the sequence?

$$162 = a_3 = a_1 r^2$$

$$18 = a_7 = a_1 r^6$$

Divide to get $r^4 = \frac{1}{9}, r = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

Beginning at the seventh term, multiply by

$$\left(\frac{\sqrt{3}}{3}\right)^3 \_or\_ \left(\frac{-\sqrt{3}}{3}\right)^3$$

$$18\left(\frac{\sqrt{3}}{3}\right)^3 = 2\sqrt{3}, 18\left(\frac{-\sqrt{3}}{3}\right)^3 = -2\sqrt{3}$$

3. The 257th term of an arithmetic sequence is 15 less than the square of the 22nd term. If the 7th term is 4, find all possible values for the common difference.

$$a_1 + 6d = 4, a_1 = 4 - 6d$$

$$(a_1 + 21d)^2 - 15 = (a_1 + 256d)$$

$$(4 - 6d + 21d)^2 - 15 = (4 - 6d + 256d)$$

$$(4 + 15d)^2 - 15 = (4 + 250d)$$

$$16 + 120d + 225d^2 - 15 = 4 + 250d$$

$$225d^2 - 130d - 3 = 0$$

$$(5d - 3)(45d + 1) = 0$$

$$d = \frac{3}{5}, \frac{-1}{45}$$

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Match 5 Team Round

1.) $\frac{63}{16}$

2.) $-\frac{12}{13}$

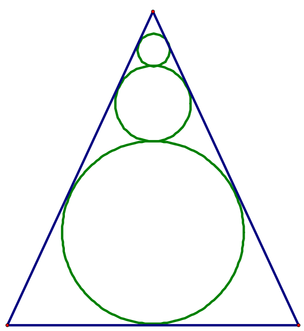
3.) $\frac{16}{x^3y^{17}}$

4.) $i, -i$

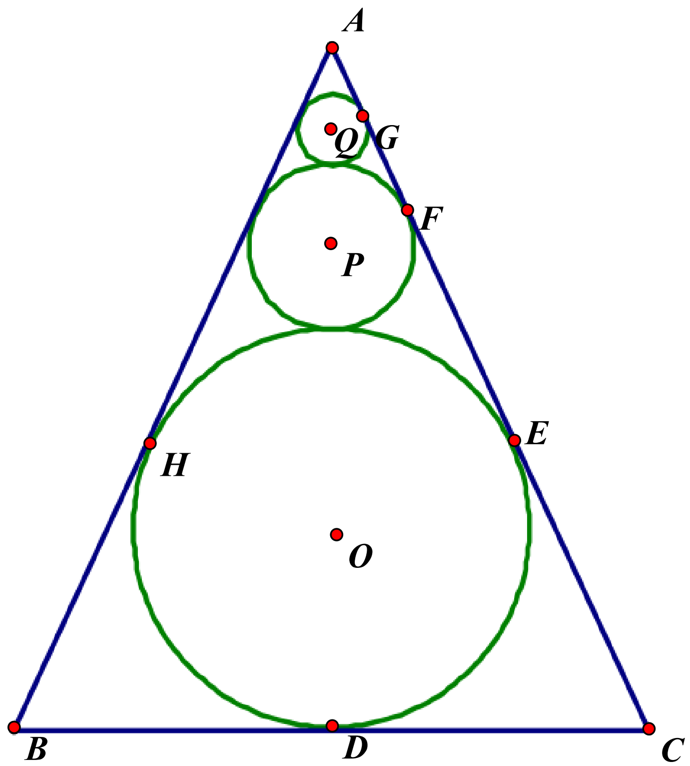
5.) $\frac{1+3\sqrt{5}}{8}$

6.) 92.16

Note: Diagrams not necessarily drawn to scale.

1.) 

1. The snowperson above is such that the circles are externally tangent to each other as shown and tangent to the sides of an isosceles triangle whose sides measure 10 cm, 10 cm, and 12 cm. Find the sum of the radii of the 3 circles.



Let the vertex angle be A, the base angles be B and C, and the centers of the circles be O, P, and Q. Draw the altitude $\overline{AD}$. Since it intersects the base at its midpoint, the altitude creates two right triangles with hypotenuse 10 cm and one base 6 cm, so AD by Pythagorean theorem is 8 cm. The lengths BD, DC, EC,and BH must all be 6 cm. . Draw a segment from the center of the large circle to one of the points of tangency E on one of the 10 cm sides. If the radius of the large circle is R, OE= R. Since OD=R, then AO=8-R. Since CE=6 and AC=10, then AE=4, so ΔAOE is a right triangle, where

$$R^2 + 4^2 = (8 - R)^2$$

$$R^2 + 16 = 64 - 16R + R^2$$

$$16R = 48, R = 3.$$

The distance from the base to the top of the first circle along the altitude is therefore 6 cm, which leaves 2 cm for the altitude for the distance from the vertex of the triangle to the bottom of the second circle. Draw a segment from P to the point of tangency F.

Then ΔAPF is similar to ΔAOE, so by similar triangles,

$$\frac{2 - r}{5} = \frac{r}{3}, 6 - 3r = 5r, 6 = 8r, r = \frac{3}{4}.$$

That leaves a length of 0.5 cm from the vertex of the triangle to the bottom of the smallest circle. Say its radius is r*. Draw a segment from Q to the point of tangency G. Then ΔAQG is similar to ΔAOF, so

$$\frac{2-r}{5}=\frac{r}{3}, 6-3r=5r, 6=8r, r=\frac{3}{4}$$

$$\frac{0.5-r*}{5}=\frac{r*}{3}, 1.5-3r*=5r*$$

$$8r*=1.5, r*=\frac{3}{16}$$

$$Sum\_is\_3+\frac{3}{4}+\frac{3}{16}=\frac{48}{16}+\frac{12}{16}+\frac{3}{16}=\frac{63}{16}$$

2.) Solve the equation $1+\cfrac{2}{3+\cfrac{4}{5+\cfrac{6}{x}}}=7$

$$1+\frac{2}{3+\frac{4}{5+\frac{6}{x}}}=7$$

$$1+\frac{2}{3+\frac{4}{\frac{5x+6}{x}}}=7$$

$$1+\frac{2}{3+\frac{4x}{5x+6}}=7$$

$$1+\frac{2}{\frac{3(5x+6)+4x}{5x+6}}=7$$

$$1+\frac{2}{\frac{19x+18}{5x+6}}=1$$

$$1+\frac{10x+12}{19x+18}=7$$

$$\frac{19x+18+10x+12}{19x+18}=7$$

$$\frac{29x+30}{19x+18}=7$$

$$29x+30=133x+126$$

$$104x=-96$$

$$x=\frac{-12}{13}$$

3.) If x≠0 and y≠0, express in simplest form with no negative exponents:

$$\frac{(2x)^6(3y^{-5})(4x^3y)^{-4}}{(9y^7)^2(12xy^2)^{-3}}$$

$$=\frac{2^6x^6*3y^{-5}*4^{-4}x^{-12}y^{-4}}{9^2y^{14}*12^{-3}x^{-3}y^{-6}}$$

$$=\frac{2^6x^6*3*12^3x^3y^6}{y^59^2y^{14}*4^4x^{12}y^4}=$$

$$\frac{2^6x^6*3*4^3*3^3x^3y^6}{y^59^2y^{14}*4^4x^{12}y^4}=$$

$$\frac{2^62^6x^6*3*3^3x^3y^6}{y^5*3^4*y^{14}*2^8x^{12}y^4}$$

$$=\frac{16x^9y^6}{x^{12}y^{23}}=\frac{16}{x^3y^{17}}$$

4.) A geometric sequence $\{a_n\}$ of complex numbers has $a_2 = -1+i$ and $a_4 = -2-2i$ Find all possible values for a_1

$$r^2 = \frac{a_4}{a_2} = \frac{-2-2i}{-1+i} = \frac{(-2-2i)(-1-i)}{(-1+i)(-1-i)} = \frac{4i}{2} = 2i$$

$$r = \sqrt{2i} = a+bi$$

$$0+2i = a^2 - b^2 + 2abi$$

$$a = 1\_and\_b = 1\_or\_a = -1\_and\_b = -1$$

$$r = a+bi = 1+i\_or\_-1-i$$

$$a_1 = \frac{-1+i}{1+i}\_or\_\frac{-1+i}{-1-i} =$$

$$\frac{(-1+i)(1-i)}{(1+i)(1-i)}\_or\_\frac{(-1+i)(-1+i)}{(-1-i)(-1+i)} =$$

$$\frac{2i}{2}\_or\_\frac{-2i}{2} = i, -i$$

5.) If $\cos(x+\frac{\pi}{3})=\frac{1}{4}$, and x is in quadrant I, what is cos(x) ?

$$\cos(x+\frac{\pi}{3})=\frac{1}{4}$$

$$\cos(x)\cos(\frac{\pi}{3})-\sin(x)\sin(\frac{\pi}{3})=\frac{1}{4}$$

$$\frac{1}{2}\cos(x)-\frac{\sqrt{3}}{2}\sin(x)=\frac{1}{4}$$

$$2\cos(x)-2\sqrt{3}\sin(x)=1$$

$$2\cos(x)-1=2\sqrt{3}\sin(x)$$

$$4\cos^2(x)-4\cos(x)+1=12\sin^2(x)$$

$$4\cos^2(x)-4\cos(x)+1=12(1-\cos^2(x))$$

$$16\cos^2(x)-4\cos(x)-11=0$$

$$\cos(x)=\frac{4\pm\sqrt{16+704}}{32}=\frac{4\pm\sqrt{720}}{32}=\frac{4\pm12\sqrt{5}}{32}=\frac{1\pm3\sqrt{5}}{8}$$

Since x is in quadrant I, take the positive value

$$\frac{1+3\sqrt{5}}{8}$$

6) $\sum_{n=1}^{\infty} a_n = 2500$ and $\{a_n\}$ is a geometric sequence of real numbers. If $a_2 = a_1 - 4$, what are all possible values for a_3?

$$a_2 = a_1 - 4$$

$$\frac{a_1}{1-r} = 2500, a_1 = (1-r)*2500$$

$$a_2 = a_1 - 4 = a_1 r$$

$$(1-r)*2500 - 4 = (1-r)*2500*r$$

$$2500 - 2500r - 4 = 2500r - 2500r^2$$

$$2500r^2 - 5000r - 2496 = 0$$

$$625r^2 - 1250r - 624 = 0$$

$$(25r - 24)(25r - 26) = 0$$

$$r = \frac{24}{25}\_or\_\frac{26}{25}$$

But if r= $\frac{26}{25}$, the series does not converge, so r= $\frac{24}{25}$.

$$\frac{a_1}{1-\frac{24}{25}} = 2500$$

$$\frac{a_1}{0.04} = 2500$$

$$a_1 = 100, a_2 = 0.96(100) = 96,$$

$$a_3 = (0.96)(96) = 92.16$$