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FAIRFIELD COUNTY MATH LEAGUE (FCML) 20~~13~~-20~~14~~

Match 5 Round 1 Algebra I: Fractions and Exponents

1.) 9

2.) $\dfrac{576x^2+8x^3-9}{9x}$

3.) $\dfrac{24yz^2-48z^{10}+20y^{18}-7y^{10}z^3}{12y^6z^8}$

1) Express as an integer or a reduced fraction:

$$(225)^5(75)^{-4}(45)^{-3}(15)^2(5)^{-1}(3)^0$$

2) If $x = a^{12}b^{-18}c^{24}$, where $a \neq 0$, $b \neq 0$, $c \neq 0$, express as a single reduced rational expression in terms of only x and constants:

$$\left(2a^2b^{-3}c^4\right)^6 + \left(a^6\right)^4\left(\frac{1}{3}b^{-18}\right)^2\left(2c^{16}\right)^3 - \left(2a^{-6}\right)^2\left(\frac{1}{4}b^6\right)^3\left(4c^{-12}\right)^2$$

3) If $y \neq 0$ and $z \neq 0$, express as a single reduced rational expression with no negative exponents:

$$\frac{2}{y^5z^6} - \frac{4}{\left(y^3\right)^2 z^{-2}} + \frac{5}{3(y^{-4})^3z^8} - \frac{7}{(2y^{-2})^2(3z)(z^5)}$$

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Match 5 Round 2
Algebra I: Fractional Expressions and Equations

1.) $-\frac{3}{8}$

2.) -2

3.) $\frac{5x+y}{y-5x}$

1). Simplify the product as much as possible if no values of x make any denominators equal to zero:

$$\frac{x^2-10x+24}{32-2x^2} * \frac{3x^2+30x+72}{4x^2-144}$$

2). Solve for all possible values of x:

$$3-\frac{22}{x+5}=\frac{6x-1}{2x+7}$$

3). Simplify as much as possible given that x≠0, y≠0, 5x≠y, and 5x≠-y

$$\frac{5xy+y^2+\dfrac{25x^2-y^2}{1-\dfrac{y}{5x}}}{y^2-5xy-\dfrac{25x^2-y^2}{1+\dfrac{y}{5x}}}$$

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Match 5 Round 3 Geometry: Circles

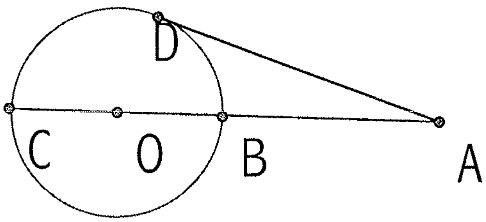
1.) 6

2.) 11

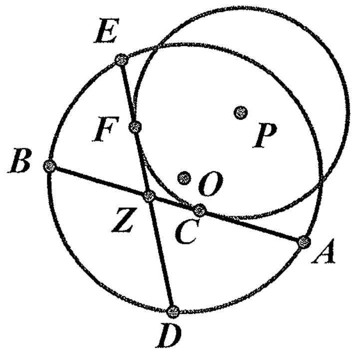
Note: Diagrams not necessarily to scale.

3.) $4\sqrt{2}-4$

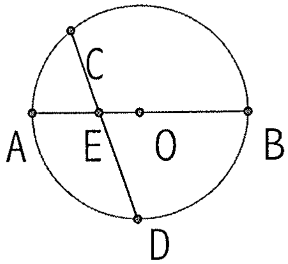
1). $\overline{BC}$ is a diameter of circle O. The line containing $\overline{BC}$ passes through point A outside the circle. $\overline{AD}$ is tangent to circle O. If AD=$\sqrt{60}$ and OC=2, find the length of $\overline{AB}$.



2). Circle P has its center in the interior of Circle O. $\overline{AB}$ and $\overline{DE}$ are chords of Circle O that meet at Z. $\overleftrightarrow{ZF}$ and $\overleftrightarrow{ZC}$ are tangent to Circle P at points F and C respectively. If AZ=$x-5$, BZ=$\frac{x}{5}$, EZ=$x-9$, DZ=$\frac{x}{4}$, and ZC=5, find the length of $\overline{EF}$.



3.) Circle O has radius 4 cm $\overline{AB}$ is a diameter of the circle and intersects chord $\overline{CD}$ at E. The length of arc AC is π cm and the measure of $\angle AEC$ is 67.5 degrees. Find the length of $\overline{OE}$ in cm.



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Match 5 Round 4 Quadratic Equations and Complex Numbers

1.) $\pm 3i, \pm 2i$

2.) $\frac{1}{3z+1}, -1$

3.) $2.5-3i, -2.5+3i$

1) If $y^2 = x$, find all complex values of y for which $x^2 + 13x + 36 = 0$

2) Solve for w in terms of z if $z \neq \frac{-1}{3}$: $(3z+1)w^2 + (3z)w + 2 = 3$

3) Find the two complex square roots of $\frac{-11}{4} - 15i$

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Match 5 Round 5
Solving Trig
Equations

1.) $0, \frac{2\pi}{3}, \frac{4\pi}{3}$

2.) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

3.) $\frac{1 \pm 6\sqrt{2}}{10}$

1) Solve for all x if $0 \leq x < 2\pi$: $2\cos^2(x) - \cos(x) = 1$

2) Solve for x if $0 < x < \pi$: $\csc(x)\cot^2(x) = \csc^2(x) + \frac{3}{\sin(x)} - 4$

3) Find all values of sin(A) such that $\sin(A + \frac{\pi}{3}) = \frac{1}{5}$

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Match 5 Round 6
Sequences and Series

1.) 196

2.) $3, \frac{-109}{9}$

3.) 256, 4

1.) The sequence $\{L_n\}$ is defined recursively as follows: $L_1=1$, $L_2=3$, and for $n>2$, $L_n=L_{n-1}+L_{n-2}$. Evaluate $\sum_{n=1}^{9} L_n$.

2.) In an arithmetic sequence, the ninth term is 6 less than the square of the second term. If the fifth term is 11, find all possible values for the first term of the sequence.

3. For a geometric sequence $\{a_n\}$, $\sum_{n=1}^{\infty} a_n = 3125$. If $a_2 = 500$, what are all possible values for the fifth term of the sequence?

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Match 5 Team Round	

1.) (4,2), (8,2), (8,4), (9,3)

2.) $\frac{7 \pm \sqrt{85}}{6}$

3.) 9

4.) $-35-5i$

5.) $\frac{-4 \pm \sqrt{7}}{3}$

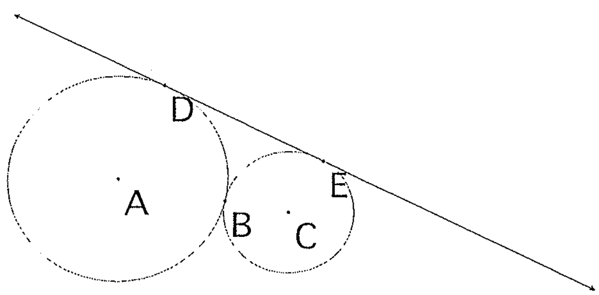
6.) $\frac{-1}{12}, \frac{16}{3}$

1) If a and b are integers such that $2 \le a \le 10$ and $2 \le b \le 10$, find all ordered pairs (a,b) such that the following expression is an integer: Give your answers as ordered pairs (a,b).

$$\frac{(77a^{2014})(13b^{-6})}{(a^{503})^4 b^{-3}} - \frac{(10a^{15})^3(b^{10})^6}{a^{43}(b^9)^7}.$$

2.) Solve this equation for all possible values of x.

Give your answers in simplest radical form. $2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{x}}} = x$.



3.) A circle with center A has radius x cm. A circle with center C has radius 5 cm. The two circles are externally tangent to one another at B. Circle A is tangent to $\overleftrightarrow{DE}$ at D and circle C is tangent to $\overleftrightarrow{DE}$ at E. The length of $\overline{DE}$ is $6\sqrt{5}$ cm. Find the radius of the circle with center A.

4) Express in a+bi form for real numbers a and b. $\dfrac{125}{(1+2i)^3} + \dfrac{625i}{(2-i)^4}$

5. Find all possible values of tan(x) such that sec(x) – 2 tan(x) = 2.

6) In a geometric sequence of complex numbers, the 5th term is 4 less than twice the third term. If the 7th term is 18, what are all possible values for the first term of the sequence?