FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 6 Round 1
Geometry: Lines and Angles

1.) 210

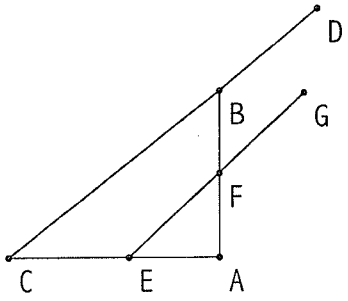
2.) -10

3.) 25, 3

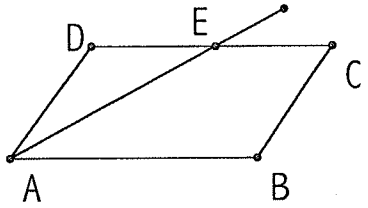
Note: Diagrams are not necessarily to scale:

1) The supplement of angle X measures 30 degrees less than three times its complement. What is the sum in degrees of the measures of the complement and supplement of angle X?

2.) ΔAFE is a right triangle with the right angle at A. The measure of $\angle FEA$ is (6y+130) degrees, and the measure of $\angle DBF$ is (120-4y). What is the value of y that would make $\overleftrightarrow{CD}$ parallel to $\overleftrightarrow{EG}$?



3.) ABCD is a parallelogram. $\overrightarrow{AE}$ bisects $\angle DAB$ and intersects $\overline{CD}$ at point E. The measure of $\angle ADE$ is z^2+30, and the measure of $\angle EAB$ is z+15. What are all possible values for the measure of $\angle DEA$ in degrees?



FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 6 Round 2
Algebra: Literal Equations

1.) $y = -\frac{4}{3}x + \frac{19}{3}$

2.) $z = \pm p, \pm \frac{9}{4p}$

3.) $q = \pm\sqrt{-3b}$

1) If $t = \frac{3-y}{4}$ and $t = \frac{2x-5}{6}$, express y in terms of x using the form y=mx+b for constants m and b.

2) If $p \neq 0$, solve for all possible expressions of z in terms of p and simplify as much as possible:
$(4p^2z^2 - 9)^2 - 25p^2z^2 = 0$

3) If $a \neq 1$ and $b < 0$, solve for all possible expressions of a in terms of b and simplify as much as possible: $5a^3 - 6(a+b)(a-2b) - 12b^2 = a^2(a-2) - 6ab + 12b$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 6 Round 3
Geometry:
Solids and
Volumes

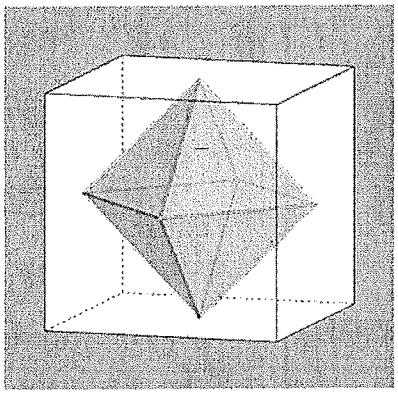
1.) $\frac{64\pi}{3}$

2.) $16\sqrt{3}$

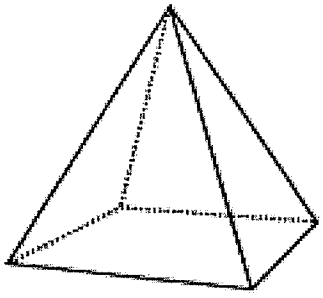
3.) $\frac{12-4\sqrt[3]{9}}{3}$

1) A spherical scoop of ice cream of radius 4 cm is placed on in a right circular cone of height 12 cm and base radius 4 cm so that half of the ice cream scoop remains outside of the cone. Find the volume of the cone that is not taken up by the ice cream. Express your answer as a single fraction.

2) An octahedron is placed inside a cube so that the 6 vertices of the octahedron meet the centers of each of the 6 sides of the cube. If the cube has volume 64 cm^3, what is the surface area of the octahedron?



3) A square pyramid has its base area 16 cm^2 and height 4 cm. A plane parallel to the base is passed through the pyramid so that the volume of the pyramid above the plane is $\frac{1}{3}$ of the volume of the original pyramid. How many cm above the base is the plane? Express your answer as a single fraction.



FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 6 Round 4
Radical Expressions and Equations

1.) $\frac{65\sqrt[3]{3}}{3}$

2.) 2

3.) 4

1) Simplify as much as possible:

$$4\sqrt[3]{375}+5\sqrt[3]{\frac{1}{9}}$$

2) What is the value of $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}}$?

3) Solve for all positive real values of x: $\sqrt{2x+10}-\sqrt{2}=\sqrt{\frac{x+12}{2}}$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 6 Round 5
Polynomials and Advanced Factoring

1.) -53

2.) $-1, 2, \pm\sqrt{3}$

3.) $(x^2+2xy+3y^2)(x^2-2xy+3y^2)$

1.) What is the remainder when $4x^3-21x^2+18x-26$ is divided by x^2-6x+9 ?

2.) Find the four real zeros of $x^4-x^3-5x^2+3x+6$

3) Factor $x^4+2x^2y^2+9y^4$ as a product of 2 polynomials with integer coefficients.

FAIRFIELD COUNTY MATH LEAGUE (FCML)

Match 6 Round 6
Counting and Probability

2013-2014

1.) 1, 3, 5, 7

2.) 64

3.) $\frac{131}{210}$

1) If r is an integer such that $0 \le r \le 8$, for which values of r is the expression $\frac{8!}{r!(8-r)!}$ a multiple of 8?

2) At Pepe's Pizza, you can get any of the following items on your pizza: Pepperoni, Sausage, Onions, Anchovies, Mushrooms, or Peppers. If the order in which the items are put on the pizza is not important and you can chose any number of toppings from 0 to 6, how many different kinds of pizza can be made?

3) 3 red balls, 4 ~~green~~ white balls, and 3 ~~white~~ green balls are placed in a container. You randomly draw 4 balls. What is the probability that you draw at least 2 red balls or at least 2 green balls? (This is not an exclusive or, so this event includes the event of drawing 2 red balls and 2 green balls.)

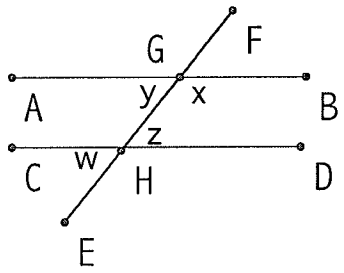
FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 6 Team Round

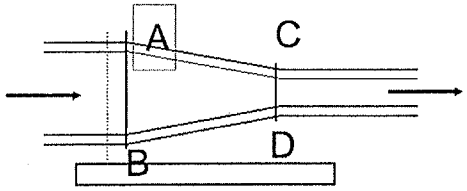
1.) $180+8m+20n$ 4.) $(a-b)^3(2a+b)^2$

2.) $\frac{37}{2}\pi$ cubic meters 5.) $\frac{23}{108}$

3.) $y=-1+x,\ y=-1-x$ 6.) 24



1) If $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$ above and $\angle FGB = (2m+5n)°$, and the angles shown are measured in degrees, express x-2y+3z+4w in terms of m and n.



2) Water flows through a pipe with a circular cross-sectional area as shown above. The diameter $\overline{AB}$ at the circular cross section to the left in the picture has length 4 meters, and the diameter $\overline{CD}$ has length 3 meters. $\overline{AC}$ and $\overline{BD}$are line segments and the distance between the centers of the two circular cross-sections shown is 6 meters. Find the volume of the part of the pipe between the circle with diameter $\overline{AB}$ and the circle with diameter $\overline{CD}$. Express your answer in cubic meters.

3.) If -1<x<1, solve for y in terms of x: $y+2=\sqrt{2y+x^2+3}$.

4.) Factor into 5 polynomials with integer coefficients:
$4a^5-8a^4b+a^3b^2+5a^2b^3-ab^4-b^5$

5.) In the game of Yahtzee, 5 evenly-balanced six-sided dice with sides labeled 1,2,3,4,5, and 6 are rolled at each turn. What is the probability that at least 3 dice show the same number in a single turn? Express your answer as a reduced fraction.

6.) In the choir, there are N sopranos, N-3 altos, N-1 tenors, and N-4 basses. They arrange themselves in a line. The members of each group may stand in any order, but all the members of each group must all stay in their section.. Let W_S, W_A, W_T, and W_B be the number of ways the sopranos, altos, tenors, and basses respectively can be arranged. If the product of W_A and W_S is 40 times the product of W_T and W_B, how many people are in the choir?