FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 6 Round 1
Geometry: Lines and Angles

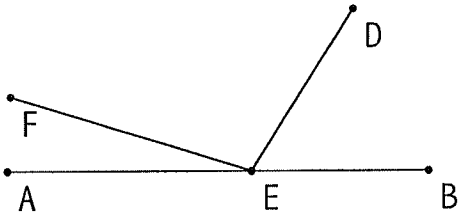
1.) 81°

2.) 150°

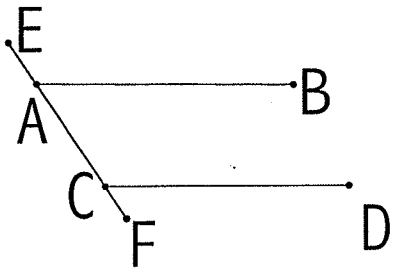
3.) $\frac{40+5x}{2}$ (5x not sqrt(x))

Note: Diagrams are not necessarily to scale:

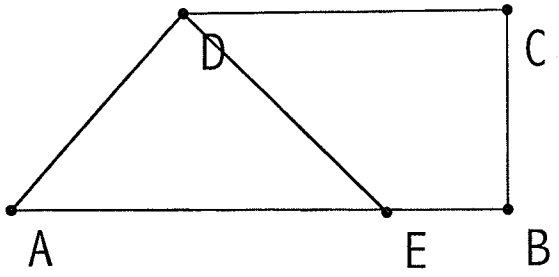
1) Point E is on line AB. $m\angle AEF$ is 9 degrees less than twice the measure of the complement of $\angle DEB$ and is one-fourth the measure of $\angle AED$. Find the measure of $\angle DEF$ in degrees.



2) Lines AB and CD are parallel and are cut by transversal AC as shown. If $m\angle BAE = (x^2+50)$ degrees and $m\angle DCF = (2x+10)$ degrees, find all possible values of $m\angle BAE$ in degrees



3) In trapezoid ABCD, segments AB and CD are parallel and AB is perpendicular to BC. A ray from D bisects angle ADC and meets side AB at E. If the measure of $\angle DAB$ is (5x-40) degrees, find the measure of $\angle BED$ in terms of x. Express your answer as a single fraction involving x.



FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 6 Round 2
Algebra: Literal Equations

1.) $a = b(y+x)$

2.) $x = -2t - \frac{7}{2}$

3.) p^2 or $\frac{p^2}{2}$

1) If $y \neq x$, solve the following for a and simplify as much as possible:
$ax + by^2 = bx^2 + ay$

2) If $y = \frac{2t-3}{4}$ and $5t = 6y - x + 1$, solve for x in terms of t.

3) Solve for all possible expressions of x in terms of p and simplify each as much as possible: $3x^2 + (2p^2)^2 - 2p^2x = 3p^4 + p^2x + x^2$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 6 Round 3
Geometry:
Solids and
Volumes

1.) $\frac{256\pi}{3}$

2.) $\frac{1000\sqrt{3}}{9}$

3.) $\frac{10\sqrt[3]{9}}{3}$

1) A spherical tennis ball of radius 4 cm is placed on an open cylindrical can of height 8 cm and base radius 4 cm so that half of the tennis ball remains outside of the can. Find the volume of the can that is not taken up by the tennis ball. Express your answer as a single fraction.

(The center of the sphere lies at the center of the circle formed by the upper rim of the cyliner.)

2) A sphere of radius 5 cm is circumscribed about a cube. Find the volume of the cube.

3) A cone is formed by rotating the line segment from (0,0) to (5,10) around the y-axis. The line y=k splits the cone so that the part of the volume of the original cone above y=k is twice the part of the volume of the original cone below y=k. Find the value of k.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 6 Round 4
Radical Expressions and Equations

1.) $\frac{26\sqrt{3}}{3}$

2.) $\sqrt[6]{5000}$

3.) 0.8 (The two solutions are 0 and 8.)

1) Express in simplest radical form: $5\sqrt{48}+\frac{30}{\sqrt{75}}-48\sqrt{\frac{25}{108}}$

2) Express $\sqrt[3]{25}$ times $\sqrt{2}$ as a single radical in the form $\sqrt[n]{a}$ for the smallest possible whole number values of n and a.

3) Solve the equation $\sqrt{10x+1}-\sqrt{2x}=\sqrt{3x+1}$ for all possible real values of x.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 6 Round 5
Polynomials and Advanced Factoring

1.) $x^3 - 6x^2 + 13x - 20$

2.) $(2a - 3b)(4a^2 + 6ab + 9b^2)$

3.) $(2a + 3b)(4a^2 - 6ab + 9b^2)$

3.) $(x^2+1)(x+8)(x-6)$

1) A cubic polynomial with integer coefficients has 4 and 1+2i as two of its zeros, where $i = \sqrt{-1}$. Express the polynomial in the form $ax^3 + bx^2 + cx + d$ for integers a,b,c, and d where a,b, c, and d are relatively prime and a>0

2) Give the complete factoring of $64a^6 - 729b^6$ as four factors with integer coefficients.

3) Give the complete factoring of the polynomial $x^4 + 2x^3 - 47x^2 + 2x - 48$ given that all factors have integer coefficients.

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Match 6 Round 6
Counting and
Probability

1.) 2

2.) 455

3.) 3744

1) Debate Club, Ecology Club, and French Club each consist of 17 students. There are 5 students who belong to both Debate Club and Ecology Club, 6 students who belong to both Debate Club and French Club, and 7 students who belong to both Ecology Club and French Club. If 21 students belong to exactly one club, how many students belong to all three clubs?

2) Fourteen different ping-pong balls numbered 1 through 14 are placed in a bin and four balls are drawn randomly. The balls are not replaced after they are drawn. How many different combinations of 4 ping-pong balls have at least 2 balls with 2 digits written on them?

3) Three cards are drawn without replacement from a standard 52-card deck of 13 different denominations and 4 different suits. Define a "half-house" as drawing 2 cards of one denomination and 1 card of a different denomination. The order in which the cards are drawn does not matter. How many different half-houses are possible?

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 6 Team Round

1.) 72 4.) $10+6\sqrt{3}$

2.) $X=a-11$ 5.) $(4a^2+5b^2+3ab)(4a^2+5b^2-3ab)$

3.) $\frac{1000\pi}{3}$ 6.) $\frac{3}{8}$

1) The measure of angle A is 3 times the square root of the measure of the complement of angle B. The measure of angle B is 3 times the measure of angle A. Find the sum of the measures of angle A and angle B.

2) Given $x\neq 2$ and $a>13$, express x in terms of a and simplify as much as possible:

$$ax^2+9x^2-4a=a^2x-81x-2a^2+198$$

3) A sphere of radius 5 cm is inscribed in a right circular cone of height 20 cm. Find the volume of the cone in cubic cm.

4) One solution to the equation $\sqrt[3]{x}=\sqrt{\frac{x+2}{3}}$ is x=1. Find the other real solution.

5) Factor into two polynomials with integer coefficients: $16a^4+31a^2b^2+25b^4$

6) A group of 4 students put their calculators into a pile, and then they each randomly choose a calculator from the pile. What is the probability that no student gets his or her own calculator?