FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 4 Round 1
Arithmetic:
Basic Statistics

1.) 248

2.) 40

3.) 0

1.) The geometric mean of the numbers $x_1, x_2, \ldots x_n$ is $\sqrt[n]{x_1 x_2 \ldots x_n}$. What is the product of the arithmetic mean and the geometric mean of the set of numbers {2, 8, 1, 16, 4} ?

2.) The interquartile range of a set of numbers is the positive difference between its upper quartile and its lower quartile. A set of 55 numbers consists of 1 10, 2 20's, 3 30's, 4 40's, and so on up to 10 100's. What is the interquartile range of this set of numbers?

3.) A set of 10 distinct numbers which have the same mean and median is arranged in ascending order. Any two numbers in the set differ by at least 10. 1 is added to the smallest number, 2 is added to the second number, 3 is added to the third number, and so on, through 10 added to the largest number. What is the difference between the new mean and the new median?

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Match 4 Round 2
Algebra 1:
Quadratic
Equations

1.) $\frac{7}{11} + \frac{13}{5}$

2.) -192

3.) $4x^2 - 73x + 144 = 0$

1.) Find the two solutions of the equation $55x^2 - 178x + 91 = 0$

2.) Find the value of k such that the equation $4(x+2)^2 - 3(x-2)^2 = k$ has exactly one solution.

3.) Find a quadratic equation whose solutions are the reciprocals of the solutions of the equation $144x^2 - 73x + 4 = 0$. Express your equation as $ax^2 + bx + c = 0$, where $a>0$ and a,b,c are relatively prime integers.

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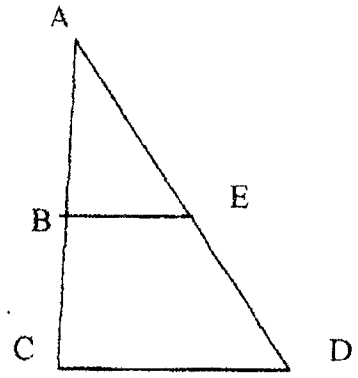
Match 4 Round 3
Geometry:
Similarity

1.) $\frac{81}{25}$

2.) $\frac{145}{6}$

3.) $\frac{12}{35}$

1.) Two regular pentagons are such that the larger pentagon has perimeter 36 and the length of one side of the smaller pentagon is 4. What is the ratio of the area of the larger pentagon to the area of the smaller pentagon (give two relatively prime integers)



2.) In the figure above, $\overline{BE} \parallel \overline{CD}$. If BC =4, ED=6, AD=10, and, BE = 3, find the perimeter of ΔACD.

3.) Right triangle ABC has sides 3,4, and 5 with the AB as the hypotenuse. The altitude drawn from C intersects AB at D. The bisector of angle ACB intersects AB at E. Find the length of $\overline{DE}$.

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Match 4 Round 4
Algebra 2:
Variation

1.) 9

2.) $240\sqrt{6}$

3.) $20\sqrt{5}$

1.) The volume V of a gas varies directly with its temperature T in Kelvins and inversely with its pressure P. A certain gas occupies 10 liters of volume at standard temperature and pressure (Standard temperature and pressure are considered 273 Kelvins and 1 atmosphere.) To the nearest liter, what is the volume of the gas at 300 Kelvins and 1.2 atmospheres?

2.) x varies directly with y, and y varies inversely with z^3. When x=5, z= $4\sqrt{3}$. What is x when z = $\sqrt{2}$?

3.) The energy in joules stored in a electrical capacitor varies jointly with its capacitance in farads and the square of the potential difference between the two plates of the capacitor in volts. If a capacitor of $3x10^{-6}$ farads has a 4 volt difference between the plates, the energy stored is 2.4×10^{-5} joules. If the same capacitor stores 0.003 joules of energy, what is the potential difference between the plates in volts?

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 4 Round 5
Trig Expressions

1.) $\sin^2 x$

2.) $-8\sin^3 x + 6\sin x$

3.) $\frac{3\sqrt{3}+88}{9}$

1.) Express in terms of a single trig function of x with no functions in the denominator:

$$\sin\left(\frac{\pi}{2}-x\right)\cos(\pi-x)\cot\left(\frac{3\pi}{2}-x\right)\tan(2\pi-x)$$

2.) Simplify and express in terms of sin x: :

$$\frac{\sec(3x)}{\csc(6x)}$$

3.) If $x = \frac{\pi}{3}$, evaluate $\cot(x) + \tan^2(3x) + \sec^3(5x) + \csc^4(7x)$

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Match 4 Round 6
Conics

1.) $\left(1, -\frac{1}{2} - \sqrt{5}\right)$

2.) $\frac{1}{2}$

3.) $16\sqrt{3}$

1) Find the location of the focus of the ellipse $9x^2 + 4y^2 - 18x + 4y - 26 = 0$ that is farthest from the origin.

2) The focus of the parabola $y=bx^2$ is 8 times farther from the origin than the focus of the parabola $x=ay^2$. If the two parabolas intersect at a point whose y-coordinate is $\frac{1}{4}$, what is the x-coordinate of this point of intersection?

3) A hyperbola with its foci at $\left(\sqrt{5},0\right) and \left(-\sqrt{5},0\right)$ passes through four points of the circle $x^2 + y^2 = 16$. One of the asymptotes of the hyperbola has slope 2. What is the area of the rectangle formed by connecting the four points of intersection?

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Match 4 Team Round

1.) $\frac{5-5\sqrt{3}}{12}$

2.) $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

3.) 12

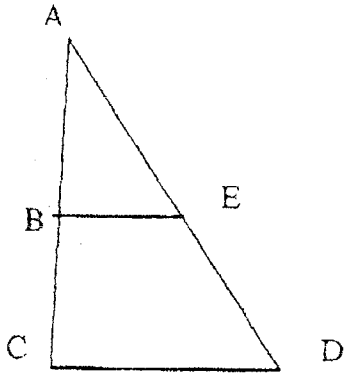
4.) $\frac{18\sqrt{5}-30}{5}$

5.) y= $\frac{\sqrt{3}}{\sin^3 x}$

6.) $\frac{2}{5}$

1.) What is the arithmetic mean of the 6 numbers $\sin(5\pi/6)$, $\cos(5\pi/6)$, $\tan(5\pi/6)$, $\cot(5\pi/6)$, $\sec(5\pi/6)$, and $\csc(5\pi/6)$?

2) Find all the solutions of the equation $\tan^2 x - \sec x - 1 = 0$ in the interval $[0,2\pi)$



3) In the figure above, BE || CD. BE is 2 more than $\frac{1}{2}$ times CD. AB is 15 less than 3 times BC. CD is twice as long as BC. Find the length of BE.

4) The asymptotes of a hyperbola are tangent to the circle $x^2 + y^2 = 9$ at the points $(2, \sqrt{5})$ and $(-2, \sqrt{5})$. If one of the y-intercepts of the hyperbola is on the circle, what is the smallest possible distance between the two y-intercepts of the hyperbola?

5) y=f(x) is defined for $0 \le x \le \frac{\pi}{2}$. y varies inversely with some power of sin(x). If $\left(\frac{\pi}{6}, 8\sqrt{3}\right)$ *and* $\left(\frac{\pi}{3}, \frac{8}{3}\right)$ both belong to the function f(x), express the function as $y = \frac{k}{(\sin x)^n}$ with numbers in place of k and n.

6) y varies directly with the square of x+1. For some value x=k, y=2. If x=k+5, then y=8. What is the median of all possible values for the constant of variation?