FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 1
Algebra I: Fractions and Exponents

1.) _____

2.) _____

3.) _____

1.) If $x = p^{-3}q^{-4}r^{-8}$ and $y = p^{6}q^{-1}r^{-1}$, then simplify and express $\left(\frac{y^{-2}}{x^{6}}\right)^{\frac{-1}{2}}$ in terms of p, q, and r using non-negative exponents.

2.) If a, b, and c are integers, then find all possible solutions (a,b,c) that satisfy the equation $2^{a}\cdot 3^{a+1}\cdot 4^{a+2}\cdot 5^{c+3}\cdot 6^{a+4}\cdot 8^{a+6}\cdot 9^{\frac{b+7}{2}}\cdot 10^{a+8} = 100$

3.) If $\left(12^{n}\cdot(-2)^{n}\cdot 4^{x}\cdot 9^{x+6}\right)^{n} = 1$, and x and n are integers, then find all real values of $\frac{x}{n}$.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 2
Algebra I: Fractional Expressions And Equations

1.) _____

2.) _____

3.) _____

1.) Simplify as a fully factored, singular fraction: $\dfrac{x}{x-\dfrac{1}{x}}+\dfrac{1}{\dfrac{1}{x}+1}$

2.) Solve for x: $\dfrac{-4}{9+x}-\dfrac{6}{2x-7}=\dfrac{2x^2-25x+6}{(x+9)(7-2x)}$

3.) Find all real values of x such that $\dfrac{\dfrac{x^2-2}{1+x}}{2-\dfrac{2}{1+x}}=\dfrac{-1+\dfrac{1}{x}}{x}$.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 3
Geometry: Circles

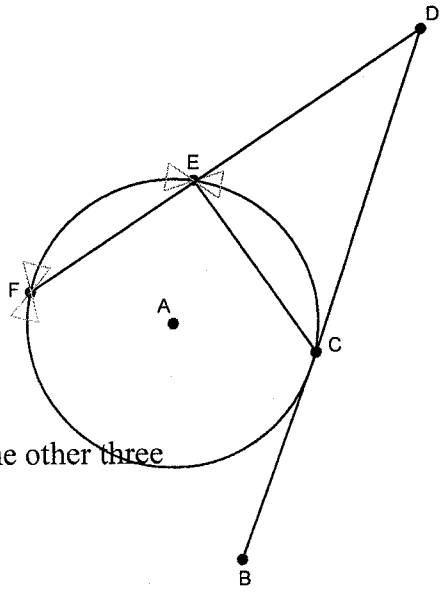
1.) _____

2.) _____

3.) _____

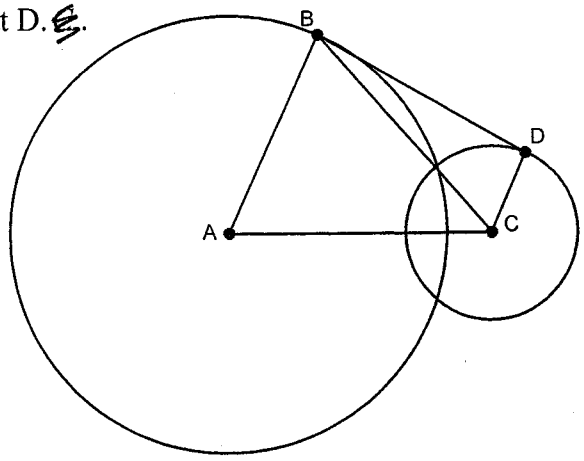
Note: Diagrams are not to scale.

1.) Segment BD is tangent to circle ~~E~~ A at ~~A~~ C. If $\overline{FE} \perp \overline{CE}$ and $m\widehat{FE} = 44°$, then find the measure of $\angle D$.



2.) Four circles of radius 2 are internally tangent to a larger circle and are each of the four are tangent to 2 of the other three circles. Find the radius of the large circle.

3.) Segment BD is tangent to circle A at B and ~~at~~ to circle C at D. If $CD = 2$, $AB = 3$, and $AC = 4$, then find BC.



FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 4
Algebra II: Quadratic Equations and Complex Numbers

1.) _____

2.) _____

3.) _____

1.) Find the distance between the points representing $1+6i$ and $-1-4i$ in the complex plane.

2.) The quadratic equation $x^2+bx+c=0$ (where b and c are complex numbers) has roots $9-4i$ and $-9+6i$. Find the product bc.

3.) For what real values of k does the equation $x^2+2(k+1)x+4=0$ have roots that are not real?

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 5
Trigonometry: Solving Trigonometric Equations

1.) _____

2.) _____

3.) _____

1.) Solve for all real values of x, given that $0 \le x < 2\pi$: $\sin\left(\frac{1}{2}x\right) = \frac{1}{2}$

2.) Solve for all real values of x, given that $-\pi \le x < \pi$: $\cos\left(2x + \frac{\pi}{2}\right) = \sin x$

3.) Solve for all real values of x: $\left(\sin^{-1}\left(\frac{x}{4}\right)\right)^2 = \frac{\pi^2}{16}$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 6
Discrete Math: Sequences and Series

1.) _____

2.) _____

3.) _____

1.) Evaluate the sum $\sum_{n=6}^{11}(n-1)(n+1)$

2.) The first four terms of an arithmetic series add up to 10. The first seven terms of the same series add up to -14. Find the 9^{th} term of the series.

3.) A sequence is defined as $s_n = an^2 + bn + c$. If the first term of the sequence is 7, the second term is 5, and the third term is 1, then find the product $a \cdot b \cdot c$.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

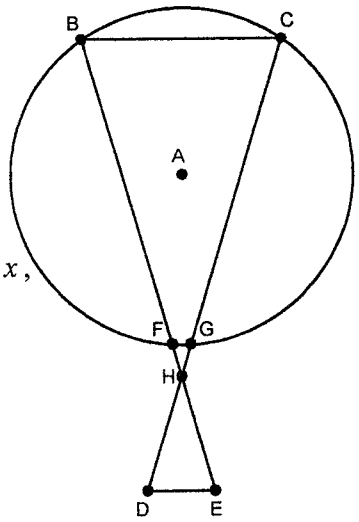
Match 5 Team Round

1.) Solve for all values of x that make the equation $2^{3^4} = (4^{3x})^x$ true.

2.) Find all real values of y such that the following equation has no solution:

$$1+\frac{x+8}{x-5}=\frac{2x^2-x-4}{(x-2y)(x-5)}$$

3.) In circle A, $CG=7$, $GH=3$, $HD=4$, $FH=\frac{5}{2}$, $BF=x$, and $HE=y$. Find all ordered pairs (x,y) such that $\Delta HBC \sim \Delta HED$.



4.) If a, and b are complex numbers, then find all possible ordered pairs (a,b) that are solutions to the system of equations:

$$\begin{aligned} 8a+8bi &= -3-8i \\ 8ai+4b &= 7-3i \end{aligned}$$

5.) Solve for all real values of x, given that $0 \le x < 2\pi$: $3\sec^2 x - 6\tan^2 x = 2\sqrt{3}\tan x$

6.) Evaluate $\sum_{j=3}^{4}\left(\sum_{i=2}^{j}\left(\sum_{n=1}^{i}\left(\frac{1}{n}\right)\right)\right)$