FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 1
Algebra I: Fractions and Exponents

1.) $\frac{1}{p^3 q^{13} r^{25}}$

2.) $(-4, -4, -5)$

3.) $\frac{-3}{2}$

1.) If $x = p^{-3}q^{-4}r^{-8}$ and $y = p^{6}q^{-1}r^{-1}$, then simplify and express $\left(\frac{y^{-2}}{x^{6}}\right)^{\frac{-1}{2}}$ in terms of p, q, and r using non-negative exponents.

2.) If a, b, and c are integers, then find all possible solutions (a,b,c) that satisfy the equation $2^{a} \cdot 3^{a+1} \cdot 4^{a+2} \cdot 5^{c+3} \cdot 6^{a+4} \cdot 8^{a+6} \cdot 9^{\frac{b+7}{2}} \cdot 10^{a+8} = 100$

3.) If $\left(12^{n} \cdot (-2)^{n} \cdot 4^{x} \cdot 9^{x+6}\right)^{n} = 1$, and x and n are integers, then find all real values of $\frac{x}{n}$.

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Match 5 Round 2
Algebra I: Fractional Expressions And Equations

1.) $\frac{x(2x-1)}{(x+1)(x-1)}$

2.) 20 or $-\frac{1}{2}$

3.) $\sqrt[3]{2}$

1.) Simplify as a fully factored, singular fraction: $\dfrac{x}{x-\frac{1}{x}}+\dfrac{1}{\frac{1}{x}+1}$

2.) Solve for x: $\dfrac{-4}{9+x}-\dfrac{6}{2x-7}=\dfrac{2x^2-25x+6}{(x+9)(7-2x)}$

3.) Find all real values of x such that $\dfrac{\frac{x^2-2}{1+x}}{2-\frac{2}{1+x}}=\dfrac{-1+\frac{1}{x}}{x}$.

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Match 5 Round 3
Geometry: Circles

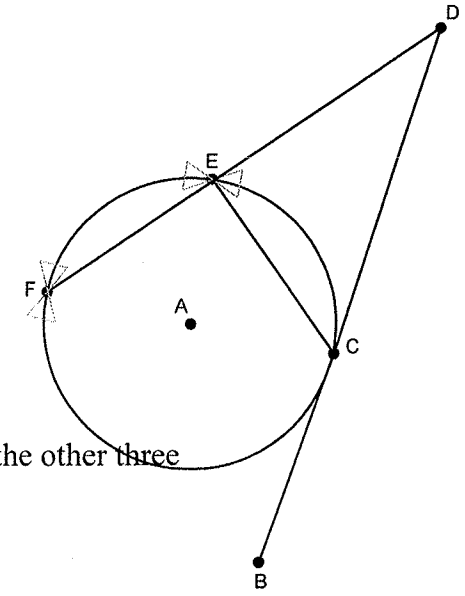
Note: Diagrams are not to scale.

1.) 22°

2.) $2+2\sqrt{2}$

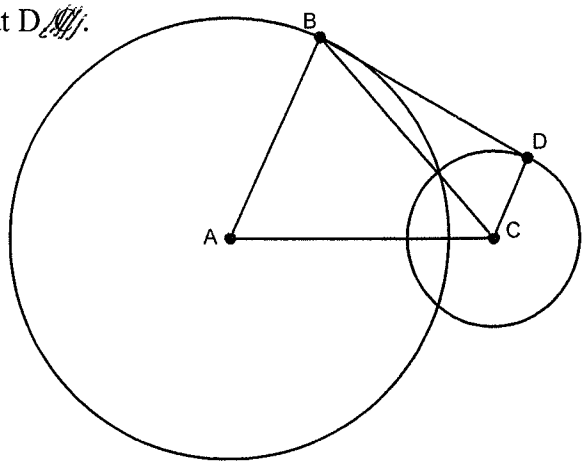
3.) $\sqrt{19}$

1.) Segment BD is tangent to circle ~~E~~ A at ~~A~~ C. If $\overline{FE} \perp \overline{CE}$ and $m\widehat{FE} = 44°$, then find the measure of $\angle D$.



2.) Four circles of radius 2 are internally tangent to a larger circle and are each of the four are tangent to 2 of the other three circles. Find the radius of the large circle.

3.) Segment BD is tangent to circle A at B and ~~at~~ to circle C at D. If $CD = 2$, $AB = 3$, and $AC = 4$, then find BC.



FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 4
Algebra II: Quadratic Equations and Complex Numbers

1.) $2\sqrt{26}$

2.) $180 + 114i$

3.) $-3 < k < 1$ [accept $(-3, 1)$]

1.) Find the distance between the points representing $1 + 6i$ and $-1 - 4i$ in the complex plane.

2.) The quadratic equation $x^2 + bx + c = 0$ (where b and c are complex numbers) has roots $9 - 4i$ and $-9 + 6i$. Find the product bc.

3.) For what real values of k does the equation $x^2 + 2(k+1)x + 4 = 0$ have roots that are <u>not</u> real?

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 5
Trigonometry: Solving Trigonometric Equations

1.) $\frac{\pi}{3}, \frac{5\pi}{3}$

2.) $-\pi, \frac{-2\pi}{3}, 0, \frac{2\pi}{3}$

3.) $\pm 2\sqrt{2}$

1.) Solve for all real values of x, given that $0 \le x < 2\pi$: $\sin\left(\frac{1}{2}x\right) = \frac{1}{2}$

2.) Solve for all real values of x, given that $-\pi \le x < \pi$: $\cos\left(2x + \frac{\pi}{2}\right) = \sin x$

3.) Solve for all real values of x: $\left(\sin^{-1}\left(\frac{x}{4}\right)\right)^2 = \frac{\pi^2}{16}$

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Round 6
Discrete Math: Sequences and Series

1.) 445

2.) −17

3.) −7

1.) Evaluate the sum $\sum_{n=6}^{11}(n-1)(n+1)$

2.) The first four terms of an arithmetic series add up to 10. The first seven terms of the same series add up to -14. Find the 9^{th} term of the series.

3.) A sequence is defined as $s_n = an^2 + bn + c$. If the first term of the sequence is 7, the second term is 5, and the third term is 1, then find the product $a \cdot b \cdot c$.
(The first sentence defines the nth term of the sequence, not the sum of the first n terms.)

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 5 Team Round

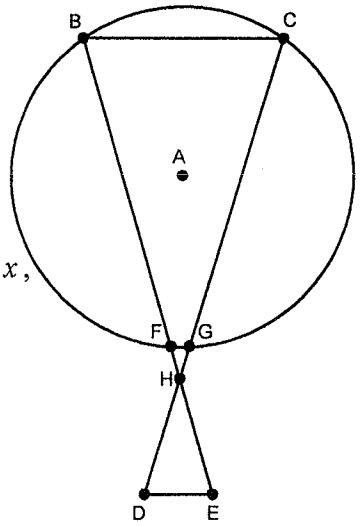
$X = \frac{\pm 3\sqrt{6}}{2}$

1.) Solve for all values of x that make the equation $2^{3^4} = \left(4^{3^x}\right)$ true.

2.) Find all real values of y such that the following equation has no solution:

$$1 + \frac{x+8}{x-5} = \frac{2x^2 - x - 4}{(x-2y)(x-5)}$$

$y \neq 1, \frac{12}{13}, \frac{1 \pm \sqrt{33}}{8}, \frac{5}{2}, \frac{-3}{4}$



3.) In circle A, $CG = 7$, $GH = 3$, $HD = 4$, $FH = \frac{5}{2}$, $BF = x$, and $HE = y$. Find all ordered pairs (x, y) such that $\Delta HBC \sim \Delta HED$.

$\left(+\frac{19}{2}, \frac{24}{5}\right)$

4.) If a, and b are complex numbers, then find all possible ordered pairs (a, b) that are solutions to the system of equations:

$$8a + 8bi = -3 - 8i$$
$$8ai + 4b = 7 - 3i$$

$\left(-\frac{3}{8} - \frac{11}{12}i, -\frac{1}{12}\right)$

5.) Solve for all real values of x, given that $0 \leq x < 2\pi$: $3\sec^2 x - 6\tan^2 x = 2\sqrt{3}\tan x$

$X = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{2\pi}{3}, \frac{5\pi}{3}$

6.) Evaluate $\sum_{j=3}^{4}\left(\sum_{i=2}^{j}\left(\sum_{n=1}^{i}\left(\frac{1}{n}\right)\right)\right)$

$\frac{35}{4}$