

April 4, 2018

Round 1: Arithmetic and Number Theory

1. (1 point) If you need to mix $\frac{1}{3}$ cup of juice concentrate with $2\frac{1}{2}$ cups of water to make a certain juice drink, how much concentrate would be necessary to make 8 cups of the same juice drink?

2. (2 points) If A, B, C, D are positive integers and $\frac{55}{24} = A + \frac{1}{B + \frac{1}{C + \frac{1}{D}}}$,

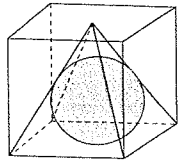
then compute $A + B + C + D$.

3. (3 points) What is the remainder when 3^{2018} is divided by 21?

1) _____

2) _____

3) _____



April 4, 2018

Round II: Algebra I, (Real numbers and no transcendental functions)

1.(1 point) Simplify:

$$\frac{(x-y)^2 - z^2}{x^2 - (y-z)^2}$$

2. (2 points) Solve for y ($b \neq 0$):

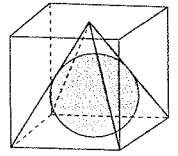
$$2b(y+ab) - 5a(b+b^2) = 3b(ab-y) - 3ab(2b+5).$$

3. (3 points) If $x \Delta y = 2x^2 - 4y$, determine (in simplest form) $(x \Delta 2) \Delta y$.

1) _____

2) _____

3) _____

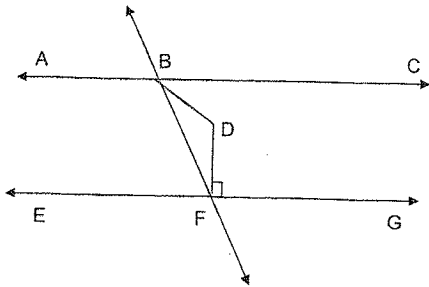


April 4, 2018

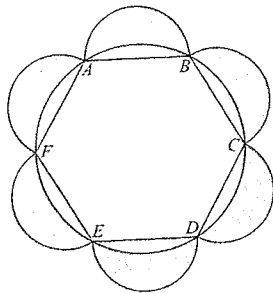
Round III: Geometry (figures are not to scale)

1. (1 point) Two similar cones have volume 54 and 128, respectively. The surface area of the smaller cone is $13\frac{1}{2}$. Find the surface area of the larger cone.

2. (2 points) In the diagram \overleftrightarrow{BF} is transversal to parallel lines \overleftrightarrow{AC} and \overleftrightarrow{EG} . $\overleftrightarrow{DF} \perp \overleftrightarrow{AB}$, \overleftrightarrow{BD} bisects $\angle CBF$. If $m\angle BDF = 115^\circ$, find $m\angle EFB$.



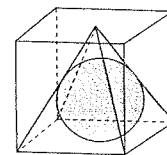
3. (3 points) Regular hexagon ABCDEF has side length 6 and is inscribed in a circle (as shown). Semicircles are constructed with AB, BC, CD, DE, EF, FA as diameters. Compute the shaded area.



1) _____

2) _____

3) _____



April 4, 2018
Round IV: Algebra II

1. (1 point) Let $f(x) = 3x + 2$ and $g(x) = x^2$. Solve the equation $g(f(x)) = 16$.

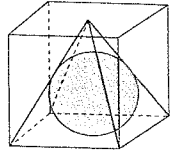
2. (2 points) Solve $e^{4x} + 16 = 13e^{2x} - 20$.

3. (3 points) Solve $\log_{13}(x^3 - 1) + \log_{\frac{1}{13}}(x - 1) = 1$.

1) _____

2) _____

3) _____



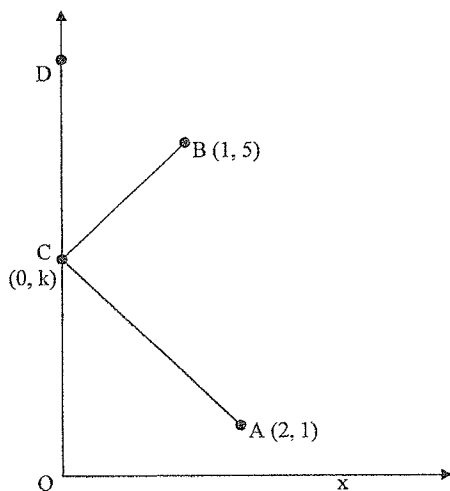
April 4, 2018
Round V: Analytic Geometry

1. (1 point) Find the value of k so that the given lines are parallel.

$$\begin{cases} 3x - ky = -4 \\ (k - 2)y - 2x = 6 \end{cases}$$

2. (2 points) Find the equation of the ellipse with foci at $(1, 3)$ and $(-1, 3)$ and with the minor axis of length 5.

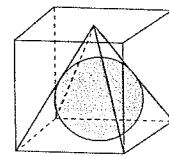
3. (3 points) Let A, B, C, D be the points shown in the diagram. Find k so that $\angle BCD \cong \angle ACO$.



1) _____

2) _____

3) _____



April 4, 2018

Round VI: Trigonometry, Complex Numbers

1. (1 point) Evaluate: $\sin 15^\circ + \cos 15^\circ$.

2. (2 points) Solve for x in terms of θ (in simplest terms) if $\tan \theta (x + \cot \theta \cos \theta) = \sec \theta$.

3. (3 points) Let z be the complex number $\cos \theta + i \sin \theta$, where $0 \leq \theta < 2\pi$, and suppose that $\left(z + \frac{1}{z}\right)^2 = 1$. Find all possible values of θ .

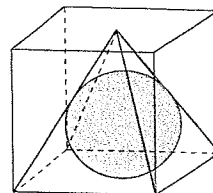
1) _____

2) _____

3) _____

April 4, 2018

TEAM ROUND



1) What 5-digit number $32a1b$ is divisible by 156?
 (a and b represent digits)

2) A student's average after 3 equally weighted test grades is 84. His first test grade is five points less than the second test grade. His third test is eight more than his second grade. How many points higher than his grade on the 3rd test would he have to score on a 4th (equally weighted test) in order to bring his average up to exactly 87?

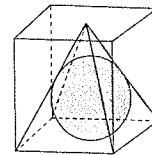
3) In triangle ABC , $AB = 4$, $BC = 5$, and $CA = 6$. Find the length of the altitude from B.

4) Given: $f(x) = \ln(e^{2x} + 2e^x)$. Write an expression for $f^{-1}(x)$ in terms of x .

5) Find the radius of the circle that has the same center as $x^2 + y^2 - 6x + 2y = 0$ and is tangent to the line $x - y - 1 = 0$.

6) If $\log_{\cos x} \sin x = \frac{1}{2}$, find the exact value of $\sec x$.

Answers:
CSAML 2018



Round 1

1) $\frac{16}{17}$ (cups)

2) 10

3) 9

Round 2

1) $\frac{x-y-z}{x+y-z}$

2) $y = -2a$

3) $8x^4 - 64x^2 + 128 - 4y$

Round 3

1) 24

2) 50 (50°)

3) $54\sqrt{3} - 9\pi$ (~~or $9(6\sqrt{3} - \pi)$~~)

Round 4

1) $\frac{2}{3}, -2$

2) $\ln 2$ or $\ln 3$

3) 3

Round 5

1) $k = 6$

2) $\frac{4x^2}{29} + \frac{4(y-3)^2}{25} = 1$ or $\frac{x^2}{\frac{29}{4}} + \frac{(y-3)^2}{\frac{25}{4}} = 1$

3) $\frac{11}{3}$

Round 6

1) $\frac{\sqrt{6}}{2}$

2) $x = \sin \theta$

3) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

TEAM

1) 32916

2) 5

3) $\frac{5\sqrt{7}}{4}$

4) $f^{-1}(x) = \ln(\sqrt{e^x + 1} - 1)$

5) $\frac{3\sqrt{2}}{2}$

6) $\frac{1+\sqrt{5}}{2}$