

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 4 Round 1  
Arithmetic:  
Basic Statistics

1.) 23.25

2.) 36

3.) 18

1.) Find the arithmetic mean of the median and the mode of the numbers  $\{13, 23, 24, 24, 15, 18, 24, 22\}$ .

2.) The geometric mean of the numbers  $x_1, x_2, x_3, \dots, x_n$  is defined to be  $\sqrt[n]{x_1 x_2 x_3 \dots x_n}$ . Find the geometric mean of the set of numbers  $\{8, 12, 36, 72, 243\}$

3.)  $\{2, 3, 5\}$  is a set of three consecutive prime numbers whose median is equal to its range. There is one other set of more than three consecutive prime numbers, all of which are between 5 and 100, such that the range of the numbers is equal to the median of the numbers. Find this number that is equal to the range and the median of this set.

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 4 Round 2

Algebra 1:

Quadratic

Equations

1.) 0, 6

2.) 9, -16

3.)  $\frac{5}{m-1}, \frac{5}{m-1}$

1.)\_ Find all solutions to the equation  $(x+3)^2 = (2x-3)^2$

2.)\_ Find all values of  $k$  such that  $(k+7)x^2 - 24x + k = 0$  has exactly one real solution.

3.)\_ If  $m \neq 1$ , give all solutions to the equation  $(m-1)x^2 + 25 = 5mx$ . Express your answers as single fractions involving  $m$ .

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Match 4  
Round 3  
Geometry:  
Similarity

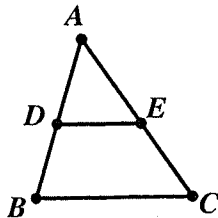
1.)  $\frac{4.8}{\quad}$

2.)  $\frac{96\sqrt{5}}{5}$  cm

3.)  $\frac{135}{\quad}$

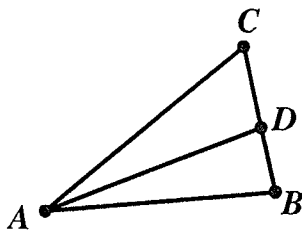
Note: Diagrams are not necessarily drawn to scale

1. In the diagram below,  $\overline{BC}$  is parallel to  $\overline{DE}$ .  $AD=6$ ,  $DB=4$ ,  $BC=8$ . Find  $DE$ .



2. The ratio of the areas of two regular octagons is 9:5. One side of the smaller octagon measures 4 cm. Find the perimeter of the larger octagon.

3. In  $\triangle ABC$  below,  $D$  lies on  $\overline{BC}$ ,  $\overline{AD}$  bisects  $\angle BAC$ ,  $AC=40$ ,  $AB=24$ , and  $AB=BC$ . Find the product of  $DB$  and  $DC$ .



# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 4 Round 4  
Algebra 2:  
Variation

1.) 9

2.) 500 pounds

3.) k=1.5 n=3

1.) z varies inversely with the cube root of (y+3). If z=6 when y=24, what is the value of z when y=5?

2. The safe load a beam can support varies jointly with the width and the square of the depth and inversely as the length. If a beam measuring 16 feet by 2 inches by 8 inches is positioned so that the width is 2 inches and the depth is 8 inches, it can support 2,000 pounds. How much weight can the same beam support if it is turned so that the width is 8 inches and the depth is 2 inches?

3. A direct power variation function has form  $y=kx^n$  where k and n are constants. Both (4, 96) and (6,324) belong to the function. Find the values of k and n.

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 4 Round 5  
Trig Expressions

1.)  $\underline{\underline{-1}}$

2.)  $\underline{\underline{-128 + 128\sqrt{3}i}}$

3.)  $\underline{\underline{4 \tan x}}$

1.) Simplify as much as possible given  $(\cos x) \neq 0$  and  $(\sin x) \neq 0$ ,  $(\sin x) \neq 1$  or  $-1$ :

$$\frac{1 - \cos^2 x}{\sin^2 x - 1} * \frac{\cot^2 x + 1}{\tan^2 x + 1}$$

2.) Express  $(1 + i\sqrt{3})^8$  as a complex number  $a + bi$ .  
(Suggestion: Use DeMoivre's theorem)

3.) Express the following in terms of  $\tan x$ , given  $\cos x \neq 0$ ,  $\sin x \neq 0$ , and  $\cos(2x) \neq 0$ :

$$(\tan x + \cot x)(\sin(2x))(\tan(2x))(1 + \tan x)(1 - \tan x)$$

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Match 4 Round 6  
Conics

1.) Center:  $(-1, 2)$  Radius:  $\sqrt{3}$

2.)  $(3, 2 + \sqrt{7})$

3.)  $(4, 6)$   $(4, -6)$   $(-\frac{34}{9}, \frac{17}{3})$   $(-\frac{34}{9}, -\frac{17}{3})$

1.) Find the center and radius of the circle  $3x^2 + 6x + 3y^2 - 12y + 6 = 0$

2.) An ellipse with major and minor axes parallel to the lines  $x=0$  and  $y=0$  centered at  $(3,2)$  and passes through  $(3,6)$  and  $(6,2)$ . Find the coordinates of the focus that is farthest away from the origin.

3.) Give the coordinates of all four points where the parabola with vertex  $(-68,0)$  that passes through the points  $(-50,3)$  and  $(-18,-5)$  intersects the two asymptotes of the hyperbola  $9x^2 - 4y^2 = 36$

**FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016 Match 4 Tm Round**  
**Diagrams not necessarily drawn to scale.**

1.)  $\frac{85}{3}$

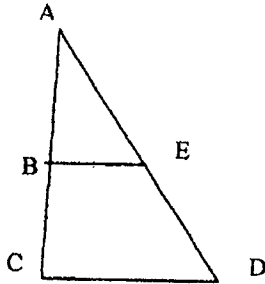
4.) 25

2.) 1.5

5.) 16

3.) 0.6

6.)  $12x^2 - 13x + 3 = 0$



1.) In the figure above  $\overline{BE}$  is parallel to  $\overline{CD}$ . If  $BE=8$ ,  $DE=x-6$ ,  $AD=2x-10$ ,  $AB=x-7$ , and  $BC=0.2x$ . What are the possible values for the perimeter of  $\triangle ACD$ ?

2.) The number of eggs laid is directly proportional to the number of chickens and inversely proportional to the time. If a chicken-and-a-half lays an egg-and-a-half in a day-and-a-half, how many eggs do 2 chickens lay in 2 days?

3.) The inter-quartile range of a set of numbers is the difference between the upper quartile (the median of the upper half of the data) and the lower quartile (the median of the lower half of the data). If there are two data points at the median value, one of them may be considered in either the lower half or the upper half of the data. A direct second power variation function has equation  $f(x)=kx^2$  for some value of  $k$  and passes through the point  $(10,4)$ . Find the inter-quartile range of the numbers  $\{f(-4), f(-3), f(-2), f(-1), f(0), f(1), f(2), f(3), f(4), \text{ and } f(5)\}$

4. Find the area of the triangle created by the two foci of the hyperbola

$$\frac{(x-1)^2}{16} - \frac{y^2}{9} = 1 \text{ and the vertex of the parabola } y = 2x^2 - 4x + 7.$$

5. If  $\sin(4x) \neq 0$ , what is the coefficient of  $\cos^4 x$  when the expression  $\frac{\sin(8x)}{\sin(4x)}$  is expressed in terms of  $\cos(x)$ ?

6. Find a quadratic equation whose solutions are the positive square roots of the solutions of  $144x^2 - 97x + 9 = 0$ . Express your answer as  $ax^2 + bx + c = 0$ , where  $a, b$ , and  $c$  are relatively prime integers and  $a > 0$ .