

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 3 Round 1
Arithmetic: Scientific
Notation and Bases

1.) 2.5×10^{-3}

2.) 1285

3.) 5, 8, 10, 11, 12

1.) Simplify and express your answer in scientific notation:

$$\frac{(4 \times 10^5)(3 \times 10^{-7})}{(8 \times 10^{-6})(6 \times 10^8)}$$

2.) Add the numbers 1232_8 , 1232_4 , and 1232_5 . Give your answer in base 9.

3.) $_b$ is a whole number base between 2 and 12 inclusive. For which values of b is the expression $\frac{1001_b}{11_b}$ NOT a prime number?

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Match 3 Round 2
Algebra: Word Problems

- 1.) 20
- 2.) 1858
- 3.) morning: 50 mph afternoon: 30 mph

1.) Susan has a collection of 50 coins, all dimes and quarters. The total value of the coins is \$9.50. How many dimes does Susan have?

2) Theodore and Franklin were two presidents. When Theodore was elected president in 1901, his age in years was 5 more than twice Franklin's age. When Theodore died in 1919, his age was 50 less than three times Franklin's age. In what year was Theodore born?
(for simplicity, assume both presidents were born on January 1.)

3.) Bill drives 10 miles from Easton to Westport in the morning. He encounters more traffic coming home so his average speed for the afternoon trip is 20 mph less than his average speed for the morning. If the total commuting time is 32 minutes, what are Bill's average speeds for each part of the trip? Give your answers in miles per hour.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 3 Round 3
Geometry: Polygons

1.) 30

2.) 9

3.) 7, 8, 9, 10

1.) The number of diagonals of a regular polygon is 405. How many sides does the polygon have?

2.) Three of the interior angles of a convex polygon add to 300 degrees. All of the other interior angles are congruent and each of these angles is two-thirds of the sum of the exterior angles associated with the three angles that add to 300 degrees. How many sides does the polygon have?

3.) The interior angle of a regular M-gon and the interior angle of a regular N-gon sum to 300 degrees. If $M < N$, there are four combinations of M and N for which this is true. Give the four possible values of M.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 3 Round 4
Algebra 2: Functions and
Inverses

Note: The inverse of a function
is not necessarily itself a function.

1.) 4

2.) $x \geq \frac{27}{4}$

3.) Domain: $[1, 10) \cup (10, \infty)$ Range: $(-\infty, -2] \cup (0, \infty)$

1.) If $f(x) = 5x+4$ and $g(x) = 2x-6$, find $g^{-1}(f^{-1}(14))$.

2.) $g(x) = x^2 - 3x + 9$. Give the domain of the relation $g^{-1}(x)$. You may either use interval notation or use the variable x in your inequality.

3.) What is the domain and range of $y = \frac{6}{\sqrt{x-1}-3}$? You may either
(a) use interval notation or: (b) use x for your inequality describing the
domain and use y for your inequality describing the range.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2015-2016

Match 3 Round 5
Advanced Math:
Exponents and Logarithms

1.) 2.498 _____

2.) $\frac{1}{25}$ + $\frac{1}{25}$ _____

3.) 5 _____

1.) If $\log_{10}(3) = 0.477$, and $\log_{10}(5) = 0.699$, and $\log_{10}(7) = .845$, find $\log_{10}(315)$

2.) Find all values of z if $\log_5(z) = y$ and $(4^{y^2+2y})(8^{y-3}) = (0.5)^{(3y+21)}$

3.) Find all values of x such that $x25^{\log_5(x+3)} - (64^{\log_4(x+2)}) = -23$

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Match 3 Round 6
Discrete Math: Matrices

1.) -39

2.) $\begin{vmatrix} \underline{17} & \underline{4} \\ \underline{-6} & \underline{-35} \end{vmatrix}$

3.) 2

1.) Give the sum of the four entries of the matrix product:

$$\begin{bmatrix} 2 & -2 & 3 \\ -4 & 5 & 1 \end{bmatrix} \begin{bmatrix} 6 & 4 \\ -3 & 2 \\ 1 & -5 \end{bmatrix}$$

2.) If $A = \begin{bmatrix} 5 & 4 \\ 6 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, find $AB^{-1} + A^{-1}B$

3.) Find all values of k such that the determinant of

$$\begin{vmatrix} k & k-4 & k+3 \\ k-1 & k & k+1 \\ 2 & -1 & 1 \end{vmatrix}$$

is -25.

FAIRFIELD COUNTY MATH LEAGUE 2015-16 Match 3 Team Round

Note: The inverse of a function or relation is not necessarily a function.

- 1.) 24 minutes 4.) 3, 30
- 2.) 1, 4, 7, A, D 5.) $(15625, 2), (\frac{1}{15625}, -1)$
- 3.) 4.9×10^{-8} seconds 6.) $27, \frac{\sqrt{3}}{27}$

1.) Working alone, Jeff can grade a stack of Math League papers in 10 minutes. Andrew can grade the same stack in 12 minutes. They work together for 3 minutes when Michael joins in and begins helping. They finish the task 2 minutes later. How long would it take Michael to grade the stack of papers alone? Give your answer in minutes.

2.) In the hexadecimal system, A=10, B=11, C=12, D=13, E=14, and F=15. Ax and xD are two-digit hexadecimal numbers. Find all hexadecimal digits x such that the expression $Ax + xD - x^2$ is divisible by 3.

3.) Spontaneous fission of a uranium nucleus which is at rest occurs. An alpha particle is emitted to the east at 2.0000×10^7 meters/second. The heavier thorium nucleus recoils to the west at 3.4×10^5 meters/second. After how many seconds will the two particles be one meter away from each other? Give your answer in scientific notation with the mantissa rounded to one decimal place.

4.) The degree measure of the sum of the interior angles of a convex n-gon is 180 more than 12 times the number of its diagonals. Find all possible values of n.

5.) If $f(x) = 0.2^{x^2 - 5x}$ and $g(x) = 25^{2x - 1}$, give the coordinates of all points where the relations $f^{-1}(x)$ and $g^{-1}(x)$ intersect.

6.) Find all values of x such that the matrix $\begin{bmatrix} 2 & 5 \\ \log_x 27 & \log_3 x \end{bmatrix}$

has determinant 1. Express any irrational values in simplest radical form.