FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 4 Round 1
Arithmetic:
Basic Statistics

1.) _____

2.) _____

3.) _____

1.)The geometric mean of the numbers $x_1, x_2, ..., x_n$ is defined to be $\sqrt[n]{x_1 x_2 ... x_n}$. What is the positive difference between the arithmetic mean and the geometric mean of the set of numbers {1, 25, 1, 5, 25} ?

2.) The upper quartile is defined to be the median of the upper half of a set of numbers. The lower quartile is defined to be the median of the lower half of a set of numbers. What is the product of the arithmetic mean, median, upper quartile, and lower quartile of the set consisting of the ten smallest prime numbers?

3.) A set of 10 numbers has a certain arithmetic mean. If you remove two numbers that add to 105, the arithmetic mean of the remaining numbers is 6 less than the arithmetic mean of the original ten numbers. What is the sum of the original 10 numbers?

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 4 Round 2
Algebra 1:
Quadratic
Equations

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1.) _____

2.) _____

3.) _____

1.) Find the two solutions of the equation $9(x+2)^2 = (4x-2)^2$.

2.) If $k \neq 0$, find all values of x in terms of k such that $kx^2 + 3 = (3k+1)x$.

3.) Find all positive integer values of m such that $m^2x^2 - 52x + \frac{96}{m} = 0$ has two rational solutions.

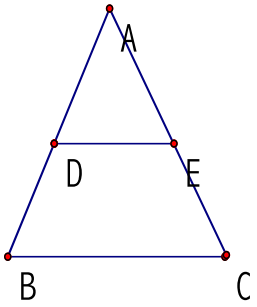
FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 4 Round 3
Geometry:
Similarity

1.) _____

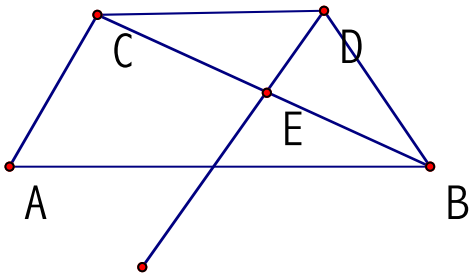
2.) _____

3.) _____cm___



1) In the figure above, $\overline{BC} \parallel \overline{DE}$. If AC=12, BC=10, DE=8, and AB=11, find the length of $\overline{BD}$.

2.) Two regular hexagons are such that the larger hexagon has area $\frac{243\sqrt{3}}{2}$ cm^2 and the perimeter of the smaller hexagon is 36 cm. What is the ratio of the lengths of the sides of the hexagons? (give two relatively prime integers; give (side of larger hexagon):(side of smaller hexagon)



3.) Isosceles trapezoid ABCD has area $16\sqrt{3}$ cm^2, The length of AC is 4 cm, and the measure of angle CAB is 60 degrees. The bisector of angle CDB intersects diagonal $\overline{BC}$ at E. Find the length of $\overline{DE}$.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 4 Round 4
Algebra 2:
Variation

1.) _____

2.) _____

3.) _____

1.) 1.) The number of people needed to do a job varies directly with the amount of work to be done and inversely with the time in which the job is to be completed. If 5 people are required to wash 20 cars in 2 hours, how many people are required to wash 192 cars in 8 hours?

2.) W varies inversely with some power of T. When T=2, W=3.75. When T=5, W=0.24. Find the function relating W and T. Express your answer as $W = \frac{k}{T^n}$, for numbers k and n.

3.) z varies jointly with the square of x and the cube of y, and y varies inversely with the fourth power of w. If z= $\frac{3}{64}$ when x=6 and w=2, what is w when z=108 and x=9?

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 4 Round 5
Trig Expressions

1.) _____

2.) _____

3.) _____

1.) Express as a single fraction:

$$\sin(\frac{2p}{3})\cos(\frac{3p}{4})\cot(\frac{5p}{6})\tan(\frac{5p}{4})$$

2.) If neither cos(x) nor sin(x) is 0, simplify and express using no other trig functions except sin(x): The middle sign is a division sign, not an addition sign.

$$\frac{\sec(x)+\tan(x)}{\csc^2(x)\tan^2(x)}, \frac{\sec^2(x)\cot^2(x)}{\sec(x)-\tan(x)} =$$

3.) If you express sin(3x)cos(4x) as a polynomial in terms of sin(x), what is the numerical coefficient of the term involving $\sin^7(x)$?

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2012-2013

Match 4 Round 6
Conics

1.) _____

2.) _____

3.) _____

1.) What is the radius of the circle $2x^2 - 10x + 2y^2 - 14y - 1 = 0$?

2.) The major axis of an ellipse is parallel to the x-axis. The length of the major axis is twice the length of the minor axis. The sum of the distances from any point on the ellipse to each focus is 8. If the ellipse passes through the points (-2,3) and (-2,-1), give the coordinates of the focus which is in the first quadrant.

3.) A hyperbola has equation $x^2-2x-2y^2+12y=7$. Find the length of the horizontal line segment that passes through one of the foci and has its endpoints on the asymptotes.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014
Match 4 Team Round

1.) _____ 4.) _____

2.) _____ 5.) _____

3.) _____ 6.) _____

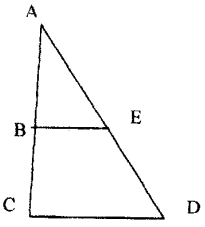
1.)If x= $\frac{p}{6}$, find the positive difference between the arithmetic mean and the median of the 6 numbers sin(x), cos(2x), cot(3x), tan(4x), csc(5x), and sec(6x).

2.) f(x) varies directly with the $\frac{3}{2}$ power of x, and g(x) varies inversely with the $\frac{2}{3}$ power of x. f(64)=g(64)=8. What is $f^{-1}(g(8))$?

3) h(x) is defined for $0 \le x < \frac{p}{2}$, and h(x) varies inversely with some power of cos(x). When x=$\frac{p}{6}$, h(x) = $4\sqrt{2}$. When x=$\frac{p}{3}$, h(x) = $12\sqrt{2}$. Express h(x) in the form h(x)= $\frac{k}{(\cos(x))^n}$ for the correct values of k and n.

4) Two of the three intersection points of the circle $x^2+y^2=25$ and the parabola y= $\frac{1}{2}x^2-5$ have the same y-coordinate. These two points are the foci of a hyperbola for which the slopes of the two asymptotes are 2 and -2. If the equation of the hyperbola is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, give the sum of the numbers a,b,h, and k given that a>0 and b>0.

5) Find all values of tan(x) such that tan(3x)=5 tan(x).



6.) In the figure above $\overline{BE}$ is parallel to $\overline{CD}$. BE=10, DE=2x+5, AD=15x, AB=14x-10, and BC=4x. What are the possible values for the perimeter of ΔACD?