FAIRFIELD COUNTY MATH LEAGUE (FCML) 2013-2014

Match 3 Round 1
Arithmetic: Scientific Notation and Bases

1.) _________3_________________

2.) __________1220_________

3.) _________-1__________________

1.) Find all possible values of the digit d such that $dd4_8$ is a multiple of 5.

0*64+0*8+4=4 NO
1*64+1*8+4 = 76 NO
2*64+2*8+4 = 148 NO
3*64+3*8+4 = 220 YES
4*64+4*8+4 = 292 NO
5*64+5*8+4= 364 NO
6*64+6*8+4= 436 NO
7*64+7*8+4=508 NO

2.) Divide the number 220_9 by the number 220_3, multiply the result by 220_4 and give your answer in base 6.

2*81+2*9=180. 2*9+2*3=24. 180/24=7.5. 2*16+2*4=40
7.5*40=300 300=1*216+2*36+2*6+0*1, so 1220

3) Find all integer values of k such that this expression gives a number between 1 and 10:

$$\frac{(2x10^{-3})^k(3x10^3)}{(9x10^{-2k})^2}$$

The terms not involving k simplify to $(3x10^3)/9^2 = 1000/27$.
The terms involving k are $2^k(10^{-3})^k/(10^{-4k}) = 20^k$.

So we need $$1<\frac{20^k*1000}{27}<10$$ $$\frac{27}{1000}<20^k<\frac{27}{100}$$ k=0 does not work, since 20^0 is not less than 27/100.

k=-1 does work. k=-2 does not work, $\frac{27}{1000}>\frac{1}{400}$ since $\frac{1}{400}=\frac{2.5}{1000}$

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Match 3 Round 2
Algebra: Word Problems

1.) __________150 _________________

2.) _________6.5_________________

3.) ___________43_________________

1.) Buford has a collection of nickels and dimes with a total value of \$10.00. Rufus has the same number of nickels as Buford has dimes, and the same number of dimes as Buford has nickels, but his collection is worth \$12.50. How many coins does each person have?

Solution: Let x = # of nickels Buford has = # of dimes Rufus has
Let y = # of dimes Buford has = # of nickels Rufus has. Solve the system
5x+10y=10.00
10x+5y=12.50
to get x=100, y = 50, so each person has 150 coins.

2.) Jesse James robs a bank at 9:00 AM and escapes toward the west at 10 miles per hour. The sheriff's office is 2 miles east of the bank. Sheriff Wyatt Earp gets the word in his office at 9:05 AM and begins his pursuit of Jesse James, traveling toward the west at 15 miles per hour. How many miles does Jesse James travel before he gets caught?

James travels 1/6 mile per minute, so when Earp starts at 9:05, James started 2 miles ahead, and has another 5/6 mile that he covered, so he has a 17/6 mile headstart. Earp travels ¼ mile per minute. If x=# of minutes from when Earp started until he catches James,

$\frac{1}{4}x = \frac{1}{6}x + \frac{17}{6}$, so $\frac{1}{12}x = \frac{17}{6}$, and x=34 minutes. In that 34 minutes, James travels $\frac{34}{6}$ miles, add that to the $\frac{5}{6}$ miles covered in the first 5 minutes , and James traveled $\frac{39}{6} = 6.5$ miles

3.) When Grant graduated from West Point, Lincoln's age was 8 less than twice Grant's age. Lee graduated from West Point 14 years earlier than Grant. When Lee graduated from West Point, Lincoln's age was 46 less than three times Lee's age. Lee was born 15 years before Grant. If Lincoln was born in 1809, how old was Grant when Lincoln died in 1865?

Let g=Grant's age when he graduated from West Point, a=Lincoln's age when Grant graduated from West Point, and r=Lee's age when Grant graduated from West Point. Then a = 2g-8, a-14=3(r-14)-46, and r=g+15. Substitute 2g-8 for a and g+15 for r into the second equation and solve: 2g-8-14 = 3(g+15-14)-46, so 2g-22=3g-43, so g=21. A=2*21-8, so a=34, and since Lincoln was born in 1809, Grant graduated in 1843 and was born in 1822. Therefore Grant was 43 when Lincoln died in 1865.

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Match 3 Round 3
Geometry: Polygons

1.) __________157.5_________________

2.) _________3__________________

3.) __________6, 12_________________

1.) A convex decagon has exactly two interior angles that are right angles. The remaining 8 interior angles are all congruent to each other. What is the degree measure of one of the 8 congruent the interior angles?

The total interior angle measure is 180(10-2) = 1440 degrees. Subtract the two right angles to get 1260 degrees. Divide 1260 by 8 to get 157.5.

2.) How many convex n-gons from n=4 through n=12 have a number of diagonals that is a multiple of 5?

N=4: 2 diagonals. N=5: 5 diagonals N=6: 9 diagonals N=7: 14 diagonals
N=8 : 20 diagonals N=9: 27 diagonals N=10: 35 diagonals N=11: 44 diagonals
N=12: 54 diagonals, so altogether N=5, N=8, N=10 work, so there are 3.

3.) Ten times the number of sides of a regular convex polygon is equal to the positive difference between the degree measures of one of its interior angles and one of its exterior angles. What are the possible values for the number of sides of the polygon?

Solve $10n + \frac{360}{n} = \frac{180(n-2)}{n}$.

$10n^2 + 360 = 180n - 360$

$10n^2 - 180n + 720 = 0$, so $n^2 - 18n + 72 = 0$, so $(n-6)(n-12) = 0$.

N=6 works since the interior angles are 120 and exterior angles are 60.

N=12 works since the interior angles are 150 and exterior angles are 30.

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Match 3 Round 4
Algebra 2: Functions and Inverses

1.) ____g(x) = $\frac{1}{4}x^2 - x - 3$________

2.) ________ $\frac{x-52}{27}$ ______________

3.) _______y>1 or y≤0____Alternately: {(-∞,0] ⋃ (1,∞)___________

1.) The inverse of g(x) is a parabola with vertex at (-4,2) that passes through the points (0,6) and (0,-2). Find g(x) in terms of x. Give your answer in the form $g(x) = ax^2 + bx + c$ for constants a, b, and c.

g(x) must be a parabola with vertex at (2,-4) that passes through (6,0) and (-2,0). The inverse of g(x) opens sideways since two values of y are matched with 0, and it is not oblique because of the symmetry. g(x) must open up or down and is a function of x, so $g(x) = a(x-2)^2 - 4$. Substitute $0 = a(6-2)^2 - 4$ to get 0 = 16a-4, so a= $\frac{1}{4}$.

Therefore, g(x) = $\frac{1}{4}(x-2)^2 - 4 = \frac{1}{4}x^2 - x + 1 - 4 = \frac{1}{4}x^2 - x - 3$.

2.) If f(x) = 3x+4, find $f^{-1}(f^{-1}(f^{-1}(x)))$. $f^{-1}(x) = \frac{x-4}{3}$.

$$f^{-1}(f^{-1}(x)) = \frac{\frac{x-4}{3} - 4}{3} = \frac{\frac{x-16}{3}}{3} = \frac{x-16}{9}$$

$$f^{-1}(f^{-1}(f^{-1}(x))) == \frac{\frac{x-16}{9} - 4}{3} = \frac{\frac{x-52}{9}}{3} = \frac{x-52}{27}$$

3.) k(5x) = $\frac{x^2}{x^2 - 25}$. What is the domain of the relation of $k^{-1}(x)$?

The domain of the inverse is the same as the range of the original function.

$$k(x) = \frac{(\frac{x}{5})^2}{\left(\frac{x}{5}\right)^2 - 25} = \frac{\frac{x^2}{25}}{\frac{x^2 - 625}{25}} = \frac{x^2}{x^2 - 625}.$$

This has vertical asymptotes at x=±25.

As x approaches -25 from the left, k(x) gets increasingly large positive. As x approaches -25 from the right, k(x) gets increasingly large negative. The same is true for x approaching 25, but in reverse. Since this is an even function, it has symmetry about the y-axis, so the part between -25 and 25 will reach its maximum in the center, where x=0. As x approaches ∞ or -∞, y approaches 1, so there is a horizontal asymptote at y=1, and the range is y>1 or y≤0

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Match 3 Round 5
Advanced Math:
Exponents and Logarithms

1.) __________64______________

2.) ____________7_______________

3.) _________ $\pm\sqrt{15}$ _______________

1.) If $\log_8 a = x$, and $\log_2 b = y$, what is the product of a and b if 1.5x+0.5y=3? (Give a simplified decimal number.)

$$ab = 8^x 2^y = (4^{1.5})^x (4^{0.5})^y = 4^{1.5x+0.5y} = 4^3 = 64$$

2.) Solve for all possible values of x: $\log_6 (0.5x+0.5) + \log_6 (x+2) = 2$

$\log_6(0.5x+0.5)(x+2) = 2$

$\log_6(0.5x^2+1.5x+1) = 2$

$0.5x^2 + 1.5x + 1 = 36$

$0.5x^2 + 1.5x - 35 = 0$

$x^2 + 3x - 70 = 0$

$(x+10)(x-7) = 0$

$x = -10, x = 7$

but -10 is extraneous, so x=7

3.) Use the approximation $\log_{10} 2 = 0.3$ to find all possible values of x such that

$$\log_2(x+5) - \log_4(x+4) = \frac{5}{3}$$

$$\log_2(x+5) = \frac{\log_{10}(x+5)}{0.3} \text{ and } \log_4(x+4) = \frac{\log_{10}(x+4)}{2\log_{10} 2} = \frac{\log_{10}(x+4)}{0.6}$$

So $\frac{\log_{10}(x+5)}{0.3} - \frac{\log_{10}(x+4)}{0.6} = \frac{5}{3}$ Multiply each side by 0.6 to get

$2\log_{10}(x+5) - \log_{10}(x+4) = 1$, so $\log_{10}\frac{(x+5)^2}{x+4} = 1$, and $\frac{(x+5)^2}{x+4} = 10^1$

so $(x+5)^2 = 10(x+4)$, so $x^2+10x+25=10x+40$, so $x^2=15$, so $x = \pm\sqrt{15}$ and both check.

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Match 3 Round 6
Discrete Math: Matrices

1.) $x = -\frac{1}{3}, \quad y = -\frac{4}{3}$

2.) -9600

3.) -1

1) Give the values of x and y that make the following true:

$$\begin{bmatrix} 5 & x \\ 2 & y \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 19 & -7 \\ 4 & -10 \end{bmatrix}$$

20+3x=19 and -5+6x=-7, so x= $-\frac{1}{3}$, and 8 +3y = 4, and -2 + 6y = -10, so y= $-\frac{4}{3}$

2) If A is the matrix $\begin{bmatrix} 5 & 4 \\ 6 & 5 \end{bmatrix}$ and B is the matrix $\begin{bmatrix} 5 & -4 \\ -6 & 5 \end{bmatrix}$

give the determinant of the matrix represented by $AB^{-1} - BA^{-1}$.

$B^{-1} = \begin{bmatrix} 5 & 4 \\ 6 & 5 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 5 & -4 \\ -6 & 5 \end{bmatrix}$, so $AB^{=1} = \begin{bmatrix} 49 & 40 \\ 60 & 49 \end{bmatrix}$ and $BA^{=1} =$ $\begin{bmatrix} 49 & -40 \\ -60 & 49 \end{bmatrix}$, so the difference is $\begin{bmatrix} 0 & 80 \\ 120 & 0 \end{bmatrix}$, so the determinant is 0-9600 = -9600

3) Give the sum of the nine entries that make up the inverse of the matrix

$\begin{bmatrix} 2 & 0 & 1 \\ \frac{1}{5} & 1 & 0 \\ 1 & 5 & 1 \end{bmatrix}$ Take the transpose to get $\begin{bmatrix} 2 & \frac{1}{5} & 1 \\ 0 & 1 & 5 \\ 1 & 0 & 1 \end{bmatrix}$

The minor for (1,1) is 1 The minor for (1,2) is -1/5 The minor for (1,3) is 0.
The minor for (2,1) is 5 The minor for (2,2) is 1 The minor for (2,3) is -10
The minor for (3,1) is -1 The minor for (3,2) is 1/5. The minor for (3,3) is 2.
Divide each entry by the determinant of the original matrix 2*1+0*(1/5)+1*0=2 to get

$\begin{bmatrix} \frac{1}{2} & \frac{5}{2} & \frac{-1}{2} \\ \frac{-1}{10} & \frac{1}{2} & \frac{1}{10} \\ 0 & -5 & 1 \end{bmatrix}$ Add the entries to get ½ + 5/2 -5 + 1 = -1

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1.) _______A.8________________ 4.) ______ $\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$ _________

2.) __________ $\frac{48}{13}$ ______hours__ 5.) ____________3, $\frac{29}{4}$ ____________

3.) _________ $\sqrt{2}$, 16___________ 6.) ___6,6,13 or 4,9,12 or 6,8,12__

1.) In the hexadecimal (base 16) system, A=10, B=11, C=12, D=13, E=14, and F=15.

Express in hexadecimal: $\frac{\left(271_{16}x10^{-7}\right)}{(19_{16}x10^{-4})^2}+\frac{(1A4_{16}x10^{-6})}{(54_{16}x10^{-5})}$

271 is 2*256+7*16+1 = 512+112+1 = 625. 19 is 1*16+9=25, so the 625 and the 25^2 cancel out, leaving only $10^{=7}/10^{-8}$ = 10 in decimal, or A in hexadecimal for the first term. In the second term 1*256+10*16+4=420, and 52=5*16+2=84, and 420/84=5, and $10^{=6}/10^{-5}=10^{-1}$, so this is 0.5 in decimal, and in hexadecimal is would be 0.8, since it would be 8/16. The answer is A.8

2.) Tommy, Ray, and Dougie are painters. If Tommy and Ray work together to paint the garage, the job takes 4 hours. If Ray and Dougie work together to paint the garage, the job takes 6 hours. If Tommy and Dougie work together to paint the garage, the job takes 8 hours. How long would the job take if all three painters worked together?

Let T=amount of time it would take Tommy to paint the garage alone
R=amount of time it would take Ray to paint the garage alone
D=amount of time it would take Dougie to paint the garage alone

$\frac{1}{R}+\frac{1}{T}=\frac{1}{4};\frac{1}{R}+\frac{1}{D}=\frac{1}{6};\frac{1}{D}+\frac{1}{T}=\frac{1}{8}$

Subtract the first two equations to get $\frac{1}{T}-\frac{1}{D}=\frac{1}{12}$, and add this to the third equation to get $\frac{2}{T}=\frac{1}{12}+\frac{1}{8}=\frac{5}{24}$, so T= $\frac{48}{5}$ hours. $\frac{5}{48}+\frac{1}{D}=\frac{1}{8}=\frac{6}{48}$, so D=48 hours. $\frac{1}{R}+\frac{1}{48}=\frac{1}{6}=\frac{8}{48}$, so R= $\frac{48}{7}$. If x is the amount of time it takes if they paint together, then $\frac{7x}{48}+\frac{x}{48}+\frac{5x}{48}=1$, so x= $\frac{48}{13}$ hours.

3.) Solve for all possible values of x:

$$(\log_4(x^2))^2 + 2 = \frac{9}{\log_x 4}$$

$\frac{1}{\log_x 4} = \log_4 x$, so this becomes $(\log_4(x^2))^2 + 2 = 9\log_4 x$, so

$(2\log_4 x)^2 - 9\log_4 x + 2 = 0$, so , $4(\log_4 x)^2 - 9\log_4 x + 2 = 0$, so

$(4\log_4 x - 1)(\log_4 x - 2) = 0$, so $\log_4 x = \frac{1}{4} or \log_4 x = 2$, so x= $4^{\frac{1}{4}} = \sqrt{2}$ or x=16

4.) If ABA = $\begin{bmatrix} 22 & -54 \\ -36 & 88 \end{bmatrix}$ and A=$\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$, find the 2x2 matrix B.

A^{-1}=$\frac{1}{-2}\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -1.5 \\ -1 & -0.5 \end{bmatrix}$. Multiply ABA by A^{-1} on the left and on the right to find B. $\begin{bmatrix} -2 & -1.5 \\ -1 & -0.5 \end{bmatrix}\begin{bmatrix} 22 & -54 \\ -36 & 88 \end{bmatrix}\begin{bmatrix} -2 & -1.5 \\ -1 & -0.5 \end{bmatrix} = \begin{bmatrix} 10 & -24 \\ -4 & 10 \end{bmatrix}\begin{bmatrix} -2 & -1.5 \\ -1 & -0.5 \end{bmatrix} =$ $\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$.

5.) $f(x)=x^2+4x-36$ and $g(x)=2x-5$. Find all values of x such that $f(g(x))-g(f(x))=g^{-1}(x)$.
$f(g(x))=(2x-5)^2+4(2x-5)-36=4x^2-20x+25+8x-20-36 =4x^2-12x-31$. $g(f(x))=2(x^2+4x-36)-5=2x^2+8x-72-5=2x^2+8x-77$. $g^{-1}(x)= \frac{x+5}{2}$, so $4x^2-12x-31-(2x^2+8x-77)= \frac{x+5}{2}$ so $2x^2-20x+46=\frac{x+5}{2}$, so $4x^2-20x+92=x+5$, so $4x^2-41x-87=0$, so $(x-3)(4x-29)=0$, so x=3 or x=$\frac{29}{4}$

6.) Three convex polygons A, B, and C have N_A, N_B, and N_C sides respectively. If the total number of diagonals of the three polygons is 83, what are all possible combinations of the three numbers N_A, N_B, and N_C (order does not matter)? Express your answers as N_A, N_B, N_C separated by the word "or".
We only need to check up to N=14, since 14(14-3)/2=77, and N=15 will have too many diagonals. Make a chart

N 3	4	5	6	7	8	9	10	11	12	13	14
D 0	2	5	9	14	20	27	35	44	54	65	77

If one polygon is a 14-gon, no combination adds to the remaining 6 diagonals. If one polygon is a 13-gon, we can have the diagonals of two hexagons add up to the remaining 18 diagonals, so one combination is 6,6,13. If one polygon is a 12-gon, we have 54+20+9=83, and 54+27+2=83, so we have two combinations 4,9,12 and 6,8,12. If one polygon is an 11-gon, no combinations of two add to the remaining 39 diagonals. If one polygon is a 10-gon, no combinations of two add to the remaining 48 diagonals. Going down more does not give us enough diagonals, so we have 6,6,13 or 4,9,12 or 6,8,12