

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 6 Round 1
 Geometry: Lines and Angles

Note: Diagrams are not to scale.

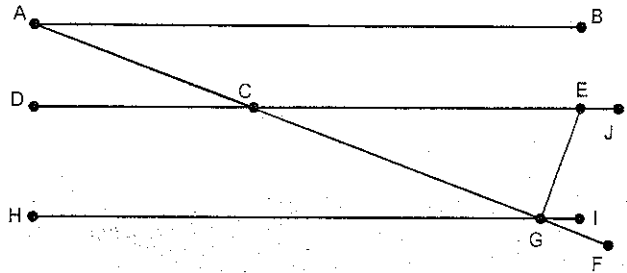
1.) 30° or 70°

2.) 105°

3.) $\frac{9}{8} \angle x \angle \frac{9}{5}$

1.) Line \overline{TR} intersects line \overline{TY} at an angle of 40° . Line \overline{TY} intersects line \overline{YR} at an angle of 110° . Find all possible measures for $\angle TRY$.

2.) Lines \overline{AB} , \overline{DJ} , and \overline{HI} are parallel.
 Lines \overline{AF} and \overline{EG} are perpendicular.
 If $m\angle ACE = x^\circ$, $m\angle JEG = y^\circ$, and
 $8x - 8y = 480^\circ$, then find $m\angle HGE$.



3.) You are given A_1 is complementary to A_2 , A_1 is supplementary to A_3 , B_1 is complementary to B_2 , and B_1 is supplementary to B_3 . If $B_3 = xA_3$ and $B_2 = 2xA_2$, then find all values of x such that $10^\circ < A_2 < 40^\circ$.

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Match 6 Round 2

Algebra I: Literal Equations

1.) $x = -4t^2 - \frac{3}{2}t + \frac{2}{3}$

2.) $x = \frac{4ay}{7(a-y)}$

3.) $b = \frac{-6a}{17}$

1.) Solve the following system of equations for x in terms of t (excluding all other variables):

$$6x + 8yt + t = 4$$
$$-y + 3t = -1$$

2.) Given that $y \neq a$, solve the following system of equations for x in terms of a and y (excluding all other variables):

$$7ax + 2by = 7xy$$
$$2a + 8b = 7b$$

3.) Given that $x \neq 0$ and $y \neq 0$, solve the following system of equations for b in terms of a (excluding all other variables):

$$8ax + 2xy = by$$
$$\frac{-7a}{y} + 5 = \frac{8b}{x}$$
$$2ay + 4xy = 2ax$$

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Match 6 Round 3
 Geometry: Solids and Volumes

Note: Diagrams are not to scale.

1.) $\frac{3}{10}$

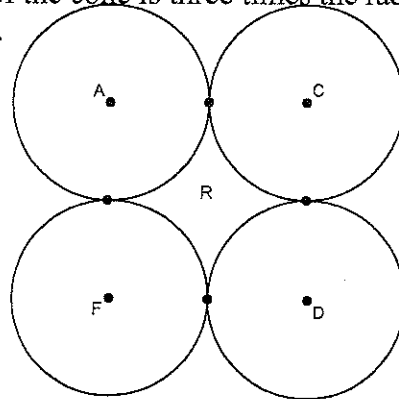
2.) $32 - 8\pi$

3.) $\frac{\sqrt{3}}{2}$

Please Note: Use the symbol π in answers that involve π . Do NOT use a decimal approximation.

1.) Cylinder A has a radius of 6 and a total surface area of 80π . Cylinder B has a radius of 6 and a volume of 80π . Find the ratio of the height of cylinder A to the height of cylinder B .

2.) The area (R) enclosed by four circles of radii 2, each tangent to two of the other circles (as shown) is the base of a cone. If the height of the cone is three times the radius of one of the circles, then find the volume of the cone.



3.) It takes one can of spray paint to cover the four faces of a right pyramid with a square base with an even coat of paint. If the base of the pyramid can be covered in the same fashion using only half a can of spray paint, then what is the ratio of the height of the pyramid to the length of one of its base edges.

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Match 6 Round 4
Algebra II: Radical Expressions
and Equations

1.) $\frac{1}{7}$

2.) $\pm \frac{1}{2}$

3.) $0, \frac{1}{81}$

1.) Evaluate: $(49)^{-\frac{1}{2}} \left(\frac{1}{7}\right)^{-8}$

2.) Solve for x : $\sqrt[3]{x + \frac{1}{2}} = \sqrt{x + \frac{1}{2}}$

3.) Solve for x : $\frac{9}{2}(16x)^{\frac{3}{4}} - (3x)^{\frac{4}{12}} - (81x)^{\frac{6}{8}} = 0$

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Match 6 Round 5

Advanced Topic: Polynomials
and Advanced Factoring

1.) $(x-1)(x-2)(x+2)(x-4)(x+4)$

2.) $a = \frac{7}{5}, -1$

3.) $\pm i\sqrt{2}$

1.) Factor $x^5 - x^4 - 20x^3 + 20x^2 + 64x - 64$

2.) If $x = -2$ is a solution of $a^2x^3 + ax^2 + a^2x + 14 = 0$, then find all possible values for a .

3.) Find all non-real solutions of $x^4 - 2x^3 - x^2 - 4x - 6 = 0$.

FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 6 Round 6

Discrete Math: Counting Arguments
and Probability

1.) $\frac{8}{243}$

2.) $\frac{16}{130}$

3.) $\frac{19}{130}$

1.) Random digits from 1 to 9 are selected for A , B , and C . What is the probability that all of the following criteria are met:

- A.) The digit B is prime.
- B.) None of the randomly selected digits is 7.
- C.) ABC is a palindrome.

Definition: Palindrome is a word or a number reading same backward as forward.

2.) Set $X = \{M, A, T, H\}$ and set $Y = \{T, E, A, M\}$. Find the number of possible sets which are subsets of X or Y but are not subsets of both X and Y .

3.) Cards with the integers from 1 to 40 are shuffled together. Nina randomly selects a card, and then Maria randomly selects a card from those that remain. What is the probability that Maria's number is within 3 of Nina's number?

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Match 6 Team Round

1.) Let us describe two angles to be *SMTC* (*slightly more than complementary*) if the sum of the measures of those angles is strictly between 90° and 100° . If $\angle A$ is SMTC with $\angle B$, $\angle B$ is SMTC with $\angle C$, $\angle C$ is SMTC with $\angle D$, $\angle D$ is SMTC with $\angle E$, $\angle E$ is SMTC with $\angle A$, and all angles are positive, then what is the largest possible difference between the maximum integer value for $m\angle C$ and the minimum integer value for $m\angle C$?

$$23^\circ$$

2.) Solve the following system of equations for y in terms of a and b .

$$x = y^2 + 2ab$$

$$y^2 = ax + by^2$$

$$y = \frac{\pm |a| \sqrt{2b(1-a-b)}}{1-a-b}$$

3.) A cube with a open top is filled with water. A sphere is pushed down into the cube until it is tangent to the five sides of cube. The displaced water pours over the top of the cube and is discarded. The remaining water in the cube is poured into an inverted cone with a base radius equal to the radius of the sphere. Find the ratio of the height of the cone to it's radius in order to have the water fill the cone exactly.

$$\frac{24 - 4\pi}{\pi}$$

4.) Solve the following inequality for all real solutions x : $\sqrt{x^2 + 2x + 1} \geq 4x + 5$

$$x \leq -\frac{6}{5}$$

5.) Factor $x^9 - x^6 + \frac{x^3}{4}$ over the real numbers.

$$\frac{x^3}{4} (x^3\sqrt[3]{2} - 1)^2 (x^3\sqrt[3]{4} + x^3\sqrt[3]{2} + 1)^2$$

$$x^3 \left(x - \frac{\sqrt[3]{4}}{2}\right)^2 \left(x^3 + \frac{\sqrt[3]{4}}{2}x + \frac{\sqrt[3]{2}}{2}\right)^2$$

6.) A deck of 28 cards consists of 6 red cards, 20 blue cards, and 2 yellow cards. A person selects 5 random cards from the deck. What is the probability that there are at least 2 red cards among the selected cards?

$$\frac{167}{585}$$