

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 4 Round 1

Arithmetic. Basic Statistics

1.)  $\frac{31}{3}$  or  $10.\bar{3}$

2.)  $\frac{21}{2}$  or  $10.5$

3.)  $12$

1.) Find the mean of: the mean, the median, and the mode of the following list of numbers  
10, 20, 0, 10, 10, 10, 20, 0, 10, 20

2.) In a sequence  $S$ , if a prime number appears then it appears once; if an even number appears then it appears twice; if a number divisible by three appears then it appears three times.

$S$  contains only the numbers 10, 7, 9, 4, 31, 15 along with the possible repetition of these numbers. Find the mean of  $a$  and  $b$ , where  $a$  is the median of  $S$  and  $b$  is the mean of  $S$ .

3.) The geometric mean of a set of numbers is defined as  $\left[ (x_1) \cdot (x_2) \cdots (x_n) \right]^{\frac{1}{n}}$  (or equivalently  $\sqrt[n]{(x_1) \cdot (x_2) \cdots (x_n)}$ ). If the arithmetic mean of  $x$ ,  $4x$ , and  $16x$  is 9 more than the geometric mean of these same numbers, then find the geometric mean of these three numbers.

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 4 Round 2

Algebra I: Quadratic Equations

1.)  $x = 1$  or  $x = 8$

2.) 6

3.)  $10x^2 \pm 27x + 18 = 0$

1.) Solve for  $x$ :  $(5x-12)(2x-2) = (3x-4)^2 - x$

2.) Find the product of all of the roots of  $(x-1)^4 - 7(x-1)^2 + 12 = 0$ .

3.) The roots of a quadratic equation are  $p$  and  $q$  such that  $p - q = \frac{3}{10}$  and  $pq = \frac{9}{5}$ . Find the equation of this quadratic in standard form.

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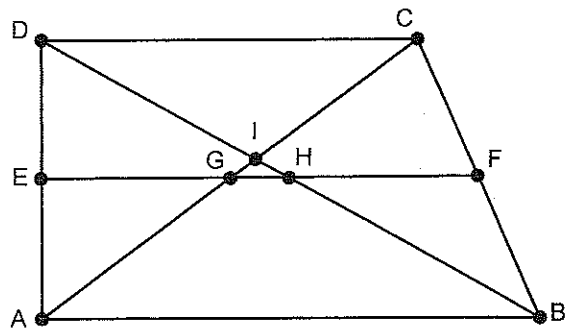
Match 4 Round 3  
Geometry: Similarity

1.)  $\frac{27}{5}$  OR 5.4

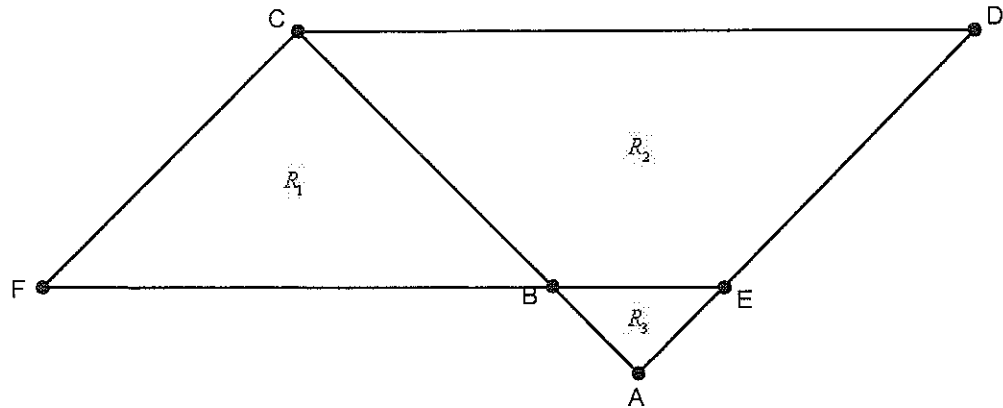
2.)  $\frac{5}{3}$

3.)  $\frac{5}{18}$

1.) Given trapezoid  $ABCD$ , with  $\overline{AB} \parallel \overline{CD}$ , median  $\overline{EF}$ ,  $AB = 18$ ,  $BH = 8$ ,  $HI = 2$ , and  $ID = 6$ , find  $EG$ .



2.) In parallelogram  $CDEF$ ,  $R_1$ ,  $R_2$ , and  $R_3$  are non-overlapping regions. If  $AB = 3$  and  $BC = 9$ , then find  $\frac{\text{Area of } R_2}{\text{Area of } R_1}$ .



3.) Cone  $A$  is similar to cone  $B$ , and cube  $C$  is similar to cube  $D$ . Cone  $A$  has radius 2 and height 3. Cone  $B$  has volume  $32\pi$ . Cube  $C$  has volume 8. Cube  $D$  has edge length 3. If the ratio of the surface area of cone  $B$  to the surface area of cube  $C$  is  $5:2$ , then find the ratio of the surface area of cone  $A$  to the surface area of cube  $D$ .

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 4 Round 4  
Algebra II: Variation

1.) 216

2.)  $9 \times 10^8$

3.)  $\frac{33}{4}$  or  $8\frac{1}{4}$

1.)  $y$  varies jointly with  $w$  and the square of  $x$ . If  $y=2$  when  $w=7$  and  $x=3$ , then find  $y$  when  $w=84$  and  $x=9$ .

2.) The gravitational force between two objects is jointly proportional to the mass of the 1<sup>st</sup> object and the mass of the 2<sup>nd</sup> object and inversely proportional to the square of the distance between them. If the gravitational force between two objects is  $4 \times 10^4$  Newtons when the objects are  $3 \times 10^4$  meters apart, then what is the gravitational force between the same two objects when they are only 200 meters apart? Write your answer in scientific notation.

3.) Suppose that  $x$  varies directly with  $y$ , and  $y$  varies inversely with  $z$ . If  $z=5\frac{1}{2}$  when  $x=3$ , and  $z=6$  when  $y=4\frac{1}{3}$ , what is the value of  $x$  when  $y=13$ ?

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 4 Round 5

Trigonometry: Simplifying and Evaluating

1.)  $-\sec^2 x$

2.)  $\csc^2 x$

3.)  $-1$

1.) Simplify the expression to a non-fractional, monomial expression involving a single

trigonometric function: 
$$\frac{\frac{\cos x}{\tan x} + \sin x}{\frac{\cos x}{\cot x} - \csc x}$$

2.) Simplify the expression to a non-fractional, monomial expression:

$$\frac{\cos^2 x \csc^2 x + (\cos x + \sin x)^2 - 2 \sin^2 x \cot x}{(\sec x + \tan x)(\sec x - \tan x)}$$

3.) In quadrilateral ABCD,  $\cos A + \cos C = 0$  and angle B is acute. Determine the value of

$$\frac{\tan B}{\tan D}$$

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

Match 4 Round 6

Coordinate Geometry: Conic Sections

1.)  $\frac{(x-2)^2}{49} - \frac{(y-2)^2}{49} = 1$

2.)  $y = \frac{3}{4}(x-5)^2 - 4$

3.)  $3\sqrt{2}$

1.) The asymptotes of a hyperbola are perpendicular to each other and pass through the center of circle  $(x-2)^2 + (y-2)^2 = 49$ . The vertices of the hyperbola lie on the same circle. If the hyperbola never intersects the  $y$ -axis, then find the equation of the hyperbola in standard form.

2.) Find the equation of a parabola if the parabola passes through the 3 vertices (located in quadrant IV) of the conic  $\frac{(x-5)^2}{4} + \frac{(y+1)^2}{9} = 1$ . Represent your answer in the form  $y = a(x-h)^2 + k$ .

3.) A circle is special case of an ellipse. As such, the area of a circle is simply a special case of the area of an ellipse. The area of an ellipse is given by  $A = \pi ab$ , where  $a$  and  $b$  are half the lengths of the major and minor axes. Find the radius of the circle, which has the same area as the ellipse  $4x^2 - 88x + y^2 - 20y + 548 = 0$ .

# FAIRFIELD COUNTY MATH LEAGUE (FCML) 2011-2012

## Match 4 Team Round

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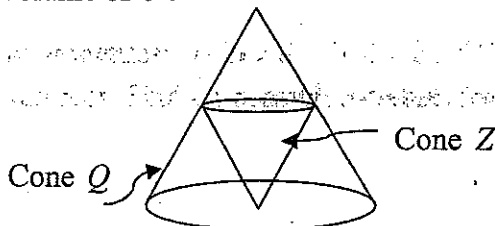
1.) A list of 8 distinct, positive integers is arranged in ascending order and has a median of 20. The median of the first three numbers in the list is 10. The median of the last three numbers in the list is twice the median of the remaining two numbers. What is the smallest possible difference between the sum of the last four numbers in the list and the sum of the first four numbers in the list?

$$\frac{14}{3} < v_x < \frac{16}{3}$$

2.) Quadratic function  $f(x)$  has the following properties:  $f(2) > 0$ ,  $f(3) < 0$ ,  $f(7) < 0$ ,  $f(8) > 0$ , and one root is three times the second root. Find all possible  $x$ -values for the vertex of  $f(x)$ .

$\frac{1}{6}$

3.) Cone  $Q$  has height  $H$  and radius  $R$ . Inverted cone  $Z$  is similar to cone  $Q$  and is located in the interior of  $Q$  with its vertex at the center of the base of  $Q$ . Additionally, every point of the circumference of the base of  $Z$  lies on the surface of cone  $Q$ . Let the region inside of  $Q$ , outside of  $Z$ , and between the planes containing the bases of the two cones be labeled as region  $J$ . Find the ratio of the volume of  $Z$  to the volume of  $J$ .



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4.)  $V$  varies directly with  $h$  and the square of  $r$ .  $h$  varies indirectly with the square of  $t$ .  $r$  varies directly with  $t$ . If  $h = 4$  when  $t = 9$ ,  $r = 12$  when  $t = 8$ , and  $V = 12$  when  $t = 4$ , then find the product of  $V$  and  $h$  when  $r = 9$ .

5.) If  $f(x, y, z) = \frac{(\csc x)(\sin y)(\csc z)}{(\tan x)(\cot y)(\cos z)}$ ,  $0^\circ < x < 90^\circ$ ,  $y = 180^\circ - x$ , and  $z = 180^\circ + x$ , then find a function  $g(x) = f(x, y, z)$  such that  $g(x)$  is a simplified, non-fractional expression in terms of  $x$ .

6.) Find all real values of  $a$  such that the conic sections  $(x-7)^2 - \frac{(y-2)^2}{a^2} = 1$  and  $y-2 = a^2(x-7)^2$  intersect.

$$\left[-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right]$$

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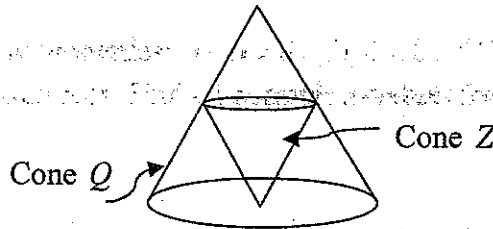
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