FAIRFIELD COUNTY MATH LEAGUE 2024-2025
Match 5
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Fractions and Exponents

1-1 Mr. Finebar has $n < 200$ peanuts. He hands one half of his peanuts to Mr. Krunchle, one third of his peanuts to Ms. Crysp, and one eighth of his peanuts to Ms. Gheefinger. If Mr. Finebar has an odd whole number of peanuts left over, find the largest possible value of n.
[Answer: 168]

1-2 There are two single digit numbers, a and b where $a > b$, that are the most probable units digits of p^q where p and q are positive integers less than or equal to 100. Find $10a + b$.
[Answer: 61]

1-3 How many zeros are found in the whole number portion of the quotient $\frac{10^{2025}+10^{24}}{10^{20}-1}$?
[Answer: 1904]

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Round 2: Rational Expressions and Equations

2-1 The expression $20+\frac{1}{24+\frac{1}{25+x}}$ is equivalent to $\frac{ax+b}{cx+d}$ for positive integers $a, b, c,$ and d, which as a set have no common factors greater than 1. What is the value of a?
[Answer: 481]

2-2 Find the smallest positive integer n such that $\sum_{k=2}^{n} \frac{2}{(2k-1)(2k-3)} > \frac{99}{100}$.
[Answer: 51]

2-3 The equation $x+\frac{6}{x-4}=\frac{x+2}{x-4}$ has extraneous solution $x=a$ and non-extraneous solution $x=b$. The equation $x^2+\frac{px^2-37x+q}{x^2-16x+r}=2x$, where p, q, and r are positive integers, has extraneous solution $x=b$ and non-extraneous solution $x=a$. Find $p+q+r$.
[Answer: 52]

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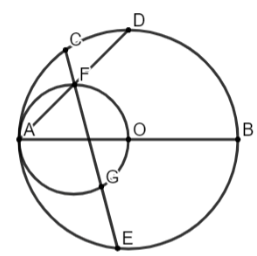
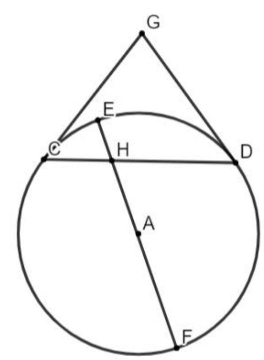
Round 3: Circles

3-1 Consider a circle with center Q and points U, A, and D on the circumference of the circle such that $QUAD$ is a convex quadrilateral. If $m\angle QUD = 20°$, find $m\angle UAD$ in degrees.
[Answer: 110]

3-2 See the diagram. Consider circle O with diameter $\overline{AB}$. Chords $\overline{CE}$ and $\overline{AD}$ meet at point F, and a circle with diameter $\overline{AO}$ intersects $\overline{CE}$ at F and G. If the length of $\widehat{AE}$ is 40π and the length of $\widehat{CD}$ is 12π, then the length of $\widehat{AG}$ is $n\pi$. Find the value of n.
[Answer: 26]



3-3 See the diagram. The circle has center A and radius r, tangents $\overline{CG}$ and $\overline{GD}$, and diameter $\overline{EF}$ which intersects chord $\overline{CD}$ at point H. If $CG = r^2 + 2025r$, $GD = 2025r + 21$, $AH = 3$, and CH and HD are integers, find the sum of all possible values of CD.
[Answer: 15]



FAIRFIELD COUNTY MATH23 LEAGUE 2024-2025
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Round 4: Quadratic Equations & Complex Numbers

4-1 If $z = 4 - 5i$ is a solution to $az^2 + bz + c = 0$, where $a, b,$ and c are integers with no common factors as a set greater than 1 and $a > 0$, find $|a| + |b| + |c|$.
[Answer: 50]

4-2 If $k = a + bi$, where a and b are positive integers, is a complex constant with the same modulus as the solutions to $z^2 + 12z + 2025 = 0$, find $a + b$.
[Answer: 63]

4-3 A quadratic function $f(z) = az^2 + bz + c$ where a, b, and c are complex constants, has the properties that $f(0) = f(3 + i) = 5 - 2i$ and $f(3 - i) = 7 - 16i$. Find $|a|^2$.
[Answer: 5]

FAIRFIELD COUNTY MATH LEAGUE 2024-2025
Match 5
Individual Section

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<u>Round 5: Trigonometric Equations</u>

5-1 If $\sin(x) = 9\cos(x)$, find $\sec^2(x)$.
[Answer: 82]

5-2 Find the sum of all possible values of $\tan(x)$ such that $4\sin^2(x) + 5 = 2025\sin(2x)$.
[Answer: 450]

5-3 If A is an angle in quadrant one such that $\sin\left(A + \frac{\pi}{6}\right) = \frac{\sqrt{6}}{4}$, then $\tan(A) = \frac{\sqrt{a} - b\sqrt{c}}{d}$, where a, b, and c are positive integers and a and c have no perfect square factors greater than 1. Find $a + b + c + d$.
[Answer: 23]

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Match 5
Individual Section

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Round 6: Sequences & Series

6-1 A sequence is defined as: $a_1 = 11$, $a_2 = 14$, and for all $n > 2$, $a_n = a_{n-1} - a_{n-2}$. What is the sum of the first 2025 terms of the sequence?
[Answer: 28]

6-2 The sum of the first two terms of an infinite geometric series is 30 and the sum of the entire series is 54. Find the sum of all possible values of the first term of the series.
[Answer: 108]

6-3 The sum of the first six terms of a decreasing arithmetic series of integers is 9. If the first term is less than 100, find the largest possible integer value of the third term in the series.
[Answer: 21]

FAIRFIELD COUNTY MATH LEAGUE 2024-2025
Match 5
Team Round

Please write your answers on the answer sheet provided.

T-1 For each rational number N there is a sequence of n strictly increasing positive integers $x_1, x_2, \dots, x_n$ such that $\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n} = N$ and each integer x_a is the smallest possible integer such that $\frac{1}{x_1} + \cdots + \frac{1}{x_a} \leq N$. If $N = \frac{2024}{2025}$, find x_5.
[Answer: 16700]

T-2 What is the sum of all single digit positive integers k such that the remainder when $x^{2025} + k$ is divided by $x - k$ has a units digit of 8?
[Answer: 13]

T-3 Consider a circle with diameter $\overline{AB}$, passing through center O, with chord $\overline{CD}$ intersecting $\overline{AB}$ at point E such that $CE: EO: ED = 1: 2: 3$. If the area of the circle is $n\pi$ where n is a positive integer less than 1000, find the largest possible integer value of CE.
[Answer: 11]

T-4 The complex number $(1 + i)^{2025} = k + ki$, where k is a positive integer. If $\log(2) \approx .301$, how many digits are in k?
[Answer: 305]

T-5 For how many positive integers N is there an angle x such that $\sec(2x) = N$ and $.36 < \sin^2(x) < .49$?
[Answer: 46]

T6 An arithmetic sequence $a_1, a_2, \dots$ contains the terms $a_n = n$ and $a_{n+1} = -1$. For how many values of n, $1 \leq n \leq 2025$, is a_1 a multiple of 5?
[Answer: 405]