FAIRFIELD COUNTY MATH LEAGUE 2024–2025
Match 2
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Factors and Multiples

1-1 If $M = lcm(18{,}24)$ and $N = lcm(12{,}45)$, find $lcm(M, N)$.

1-2 The number n has exactly 60 factors, including 1 and itself, and the largest possible number of trailing zeros. What is the smallest possible value of n?

1-3 Let k be a positive integer such that $gcf(k, 54) = 18$ and $lcm(k, 540) = 3780$. If A is the sum of all possible values of k and B is the greatest common factor of all possible values of k, find $\frac{A}{B}$.

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Round 2: Polynomials and Factoring

2-1 If $f(x) = x^2 + 3x - 1$ and $g(x) = x^2 - (f(2) - 1)x - f(11)$, find the positive zero of $g(x)$.

2-2 The polynomial $f(x) = x^3 + ax^2 + bx + 2024$ has three integer zeros, one of which is shared with the polynomial $g(x) = 3x^2 - 31x - 22$. Find the smallest possible positive value of a.

2-3 There exist positive integers $a, b,$ and c such that if the term ax^by^c is added to and subtracted from the polynomial $27x^9 + 576x^3y^6 + 512y^9$, the resulting polynomial is factorable as a difference of the cubes of two polynomials in x and y with integer coefficients. Find $a + b + c$.

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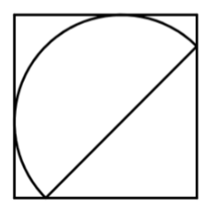
Round 3: Area and Perimeter

3-1 The difference between the area of a circle with radius k and a circle with diameter k is 1200π. Find the value of k.

3-2 A “deathly hallows” is a symbol composed of an equilateral triangle ABC, an altitude $\overline{BD}$ with D on $\overline{AC}$, and a circle inscribed in the triangle. If Harry is going to use string to make a deathly hallows such that the inscribed circle has an area of 48π square inches, then the amount of string he would need in inches to make the whole figure is $a + b\sqrt{c} + d\sqrt{e}\pi$, where $a, b, c, d,$ and e are positive integers and c and e have no perfect square factors greater than 1. Find $a + b + c + d + e$.

3-3 A semicircle with area 100π is inscribed in a square such that the diameter of the semicircle is parallel to a diagonal of the square, as shown in the figure. If the area of the square is A and the perimeter of the square is P, find $A - 5P$.



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Round 4: Absolute Value & Inequalities

4-1 What is the largest integer that satisfies the inequality $|150x - 2024| < 1130$?

4-2 The functions $f(x) = |2x - 1|$ and $g(x) = 5 - |x + 2|$ share the ordered pairs (a, b) and (c, d). Find $|a| + |b| + |c| + |d|$.

4-3 The function $f(x) = \big||3x + 2024| - 6\big| - k$, where $0 < k < 6$, has four x-intercepts. If the area enclosed by $f(x)$ between the second and third x-intercepts and bounded below by the x-axis has an area of $\frac{27}{16}$, then $k = \frac{a}{b}$ where a and b are positive integers that have no common factors greater than 1. Find $a + b$.

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Round 5: Law of Sines and Cosines

5-1 In triangle ABC, $\sin(A) = \frac{3}{5}$, $\sin(B) = \frac{1}{15}$, and $AC + BC = 140$. Find BC.

5-2 A hiker hikes 15 kilometers directly west from point A, then turns in a direction of $\alpha°$ East of South and hikes for 7 kilometers to point B. If at that point the hiker turns and walks directly back to point A, she will have to walk a distance of D kilometers from point B to point A. If $\cos(\alpha°) = \frac{3}{5}$, find D^2.

5-3 Consider triangle ABC with point D on $\overline{AC}$ such that $AD = DB$ and $CD = 2AD$. If $AB = 12$ and $\cos(\angle BDA) = \frac{1}{9}$, find BC.

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Match 2
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Round 6: Equations of Lines

6-1 The line $y = 5x + b$, where b is constant, passes through $(3,4)$. What is slope of a line that passes through $(-8,1)$ and $(b, 10b)$?

6-2 The line $3x - 5y = -10$ is reflected about the line $y = x$ to produce the function $g(x)$, and the two lines intersect at point P. The linear function $h(x)$ is perpendicular to $g(x)$ and intersects $g(x)$ at P. Find $h(-10)$.

6-3 Two linear functions, $f(x) = m_1x + b_1$ and $g(x) = m_2x + b_2$, intersect at (3,6). If $m_1 > 0$, $m_2 = -\frac{3}{2}m_1$, and the triangle formed by $f(x)$, $g(x)$, and the x-axis has an area of 37.5 square units, then $b_2 = \frac{p}{q}$ where p and q are positive integers with no common factors greater than 1. Find $p + q$.

FAIRFIELD COUNTY MATH LEAGUE 2024-2025
Match 2
Team Round

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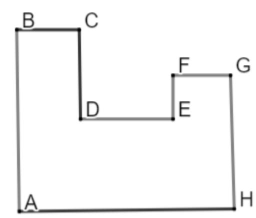
1-1 Let the set P be the set of the first 10 prime numbers. Let the set N be a set of ten positive integers with the property that for every element p in P, there is an element n in N such that the element n is the smallest positive number with exactly p factors. Find the median of the elements of N.

1-2 What is the largest positive integer value $c < 10000$ such that the quadratic $x^2 - 2025x + c$ is factorable into two binomials with integer coefficients?

1-3 In the figure shown (not necessarily drawn to scale) all angles are right angles. $BC = 3$, $FG = 2$, $GH = 10$, and $CD = DE$. If the figure has a perimeter of 66 units and an area of 127 square units, then $EF = \frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a + b$.



1-4 The inequality $a \leq |x - b| \leq c$, where $a, b,$ and c are real numbers, has the solution set $[x_1, x_2] \cup [x_3, x_4]$. The inequality $|2y - 5| \leq 7$ has solution set $[y_1, y_2]$. If $y_1 + x_2 + x_4 = x_1 + x_3 + y_2$, $x_1 = -\frac{10}{3}$, and $x_3 = \frac{9}{2}$, then $b = \frac{p}{q}$ where p and q are positive integers with no common factors greater than 1. Find $p + q$.

1-5 Consider triangles ABC and DEF which have equal areas. $AB < AC < DE < DF$, and the four values form an arithmetic sequence. If $\frac{\sin(D)}{\sin(A)} = \frac{2}{5}$, AB is an integer, and $DF < 2024$, find the largest possible value of AB.

1-6 A linear function $f(x)$ passes through the center of a circle with equation $(x - 8)^2 + (y - 21)^2 = 289$ and intersects the circle at its y-intercept. If $f(x)$ has an x-intercept of $(a, 0)$ where $a > 0$, then $a = \frac{p}{q}$ where p and q are positive integers with no common factors greater than 1. Find $p + q$.