FAIRFIELD COUNTY MATH LEAGUE 2024–2025
Match 1
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Percentages

1-1 A number x is decreased by 20% to make the number y. Then y is increased by $66\frac{2}{3}\%$ of 225% of itself. If this produces a final value of 2024, what is the value of x?

1-2 Crazy Jean's Doinkatorium is having a clearance sale: buy two doinks, get the more expensive one for 40% off and the less expensive one for 60% off. Marius finds a green doink he wants which is marked \$80, but is torn for his second doink between a spotted one which is $\$a$ and a striped one which is $\$b$. Marius notices he would save twice as much total money if he chose the spotted doink. If a and b are integers such that $b < 80 < a$, find the smallest possible value of b.

1-3 Consider the rational number $p = \frac{20}{d}$, where d is a positive integer greater than 20 and less than 100. Increasing the 20 in numerator of p by $d\%$ and the d in the denominator of p by 20% increases the value of p by $n\%$ where n is a positive integer. Find the sum of the smallest and largest possible values of d.

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Round 2: Solving Equations

2-1 Solve for x: $\sqrt{1+3\left(2-\left(5-4(1+2x)\right)\right)}=10$.

2-2 The equation $\frac{1}{x}+\frac{2}{3}=m-8$, where m is a constant, has no solutions for x when $m=p$ and a solution of $x=\frac{3}{44}$ when $m=q$. Find $p+q$.

2-3 If a and b are positive constants such that the equation $ax+21=b(3x+a)$ has infinite solutions for x, find $(a+b)^2$.

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Round 3: Triangles and Quadrilaterals

3-1 An equilateral triangle has a perimeter of k centimeters and an interior angle measure of $(3k - 21)°$. What is the length of one side of the triangle in centimeters?

3-2 If an equilateral triangle has the same perimeter as an isosceles right triangle with area 18, then the area of the equilateral triangle is $a\sqrt{b} + c\sqrt{d}$ where $a, b, c,$ and d are positive integers and b and d have no perfect square factors greater than 1. Find $a + b + c + d$.

3-3 Consider kite $ABCD$ where $AB = BC = 30$ and $m\angle A = m\angle D = m\angle C$. If the difference between the measures of the largest angle in the kite and the smallest angle in the kite is 40°, then the sum of all possible values of AC is $a + b\sqrt{c}$ where a, b, and c are positive integers and c has no perfect square factors greater than 1. Find $a + b + c$.

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Round 4: Systems of Equations

4-1 If the ordered pair (a, b) solves the system $\begin{cases} 4x + 6y = 51 \\ y = 5x \end{cases}$, find $a + b$.

4-2 If the system $\begin{cases} ax + by = 18 \\ 7x - 3y = a - 5 \end{cases}$ where a and b are constants has infinite solutions for (x, y) and $b > 0$, then $b = \frac{p}{q}$ where p and q are positive integers with no common factors greater than 1. Find $p + q$.

4-3 The system $\begin{cases} \frac{5}{x+y} + \frac{3}{x-y} = \frac{x+y}{x-y} \\ 4x - y = A \end{cases}$, where A is a constant, has solutions (x_1, y_1) and (x_2, y_2) where $x_1 > x_2$. If $x_1 + x_2 = 12$, then $y_1 = \frac{a\sqrt{b}-c}{d}$ where a, c, and d are relatively prime positive integers and b is a positive integer with no perfect square factors greater than 1. Find $a + b + c + d$.

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Round 5: Right Triangles

5-1 A spot on flat ground 2024 feet from the base of a skyscraper has an angle of elevation to the top of the skyscraper with a tangent of .75. What is the distance in feet from the spot on the ground to the top of the skyscraper?

5-2 Right triangle TRI has right angle R. If TI and RI are integers that are 5 units apart and $0 < \cot(T) < 1$, find the smallest possible value of TI.

5-3 Consider right triangle ABC with right angle B and point D on $\overline{AC}$ and point E be on $\overline{AB}$ such that $\overline{BC} || \overline{DE}$. If $\tan(\angle CAB) = \frac{3}{4}\tan(\angle DBA)$ and $\cos(\angle DBA) = \frac{2}{5}$, find the least possible integer value of AB such $(BC)^2$ is an integer.

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Round 6: Coordinate Geometry

6-1 If the graph of $f(x)$ is the perpendicular bisector of a line segment with endpoints $(1,6)$ and $(2,3)$, what is $f(27)$?

6-2 Point A has coordinates (j,k), and Point A is rotated $90°$ counterclockwise to make point B. If the midpoint of A and B is $\left(\frac{\sqrt{3}}{2}, 5\sqrt{3}\right)$, find the value of $j^2 - k^2$.

6-3 Circles with equations $(x-5)^2 + (y-9)^2 = 9$ and $(x+1)^2 + (y-1)^2 = 64$ intersect at points P and Q. $PQ = \frac{a\sqrt{b}}{c}$ where a and c are positive integers with no common factors greater than 1 and b is a positive integer with no perfect square factors greater than 1. Find $a+b+c$.

FAIRFIELD COUNTY MATH LEAGUE 2024–2025
Match 1
Team Round

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1. The positive integer k has the properties that reducing k by 20% produces an even integer, reducing k by 12.5% produces an odd integer, and while k is not a multiple of 9, increasing k by k% does produce a multiple of 9. Find the least possible value of k.

2. How many ordered pairs (x, y), where x and y are positive integers less than 100, solve the equation $5 + \frac{3y-42}{x-2y} = 2 - \frac{2x}{x-2y}$?

3. Consider parallelogram $FCML$, with $FC = ML = 10$. The altitude from vertex F intersects $\overline{LM}$ at point P and the altitude from vertex M intersects $\overline{FL}$ at point Q. If the parallelogram has an area of 50 and $MQ = \frac{4}{3}FL$, then $(LP)^2 = \frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a + b$.

4. If the ordered pair (a, b) solves the system $\begin{cases} \frac{2}{x} + \frac{4}{y} = 27 \\ 3x + 6y = 10 \end{cases}$, find the value of $\frac{a}{b} + \frac{b}{a}$.

5. A right triangle with area 12 has legs whose lengths sum to 13. The length of the altitude from the vertex of the right angle to the hypotenuse is $\frac{p}{q}$ where p and q are positive integers with no common factors greater than 1. Find $p + q$.

6. Point P on the line $y = 5x$ is reflected across $y = x$ to a point P' on the line $y = \frac{1}{5}x$. If the distance from P to P' is 8 units, find the square of the distance from the origin to P.