FAIRFIELD COUNTY MATH LEAGUE 2023–2024
Match 1
Individual Section

Please write your answers on the answer sheet provided.

Round 1: Percentages

1-1 What is the positive difference between 123% of 123 and 77% of 77?

1-2 If n is a four digit number (with no 0 digits) such that the hundreds digit is 50% larger than the tens digit and the units (ones) digit is $33\frac{1}{3}\%$ greater than the sum of the thousands digit and the tens digit, find the difference between the largest and smallest possible values of n.

1-3 Positive numbers x and y have the properties that their product is 10 and the quantity of x increased by y percent is equal to the quantity of y increased by $(2x)\%$. Find the value of $100(x^2 + y^2)$.

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Round 2: Solving Equations

2-1 Solve for x: $\frac{1}{12}x - 1 = \frac{1}{60}x + \frac{1}{15}$

2-2 If x and a are positive integers less than 100 that satisfy the equation $2(4x - 2) + a = 7(a - 1) + 3x$, what is the largest possible value of a?

2-3 If the equations a, b, c, and d are constants such that the equations $x + a = 0, x^2 + x + b = 0$, $x^3 + x^2 + x + c = 0$, and $3x^2 - 8x + 4x^3 = d$ all share a solution for x, then there exists integers P, Q, and R such that $d = Pa + Qb + Rc$. Find $P + Q + R$.

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Round 3: Triangles and Quadrilaterals

3-1 A triangle has angle measures $(2x + 32)°, (6x - 12)°$ and $(8x)°$. What is the measure of the smallest angle of the triangle in degrees?

3-2 Triangle ABC has sides of length $7, 11,$ and x. Triangle DEF is isosceles and has sides of length $7, 15,$ and x. Triangle GHI is equilateral and has the same perimeter as triangle ABC. The altitude of triangle GHI is $\frac{a\sqrt{b}}{c}$, where b has no perfect square factors greater than 1 and a and c have no common factors greater than 1. Find $a + b + c$.

3-3 A right trapezoid has a height of 20 units, an area of 360 square units, and a smaller diagonal of length 25 units. What is the length in units of the longer diagonal?

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Round 4: Systems of Equations

4-1 Bluebird Orchards sells donuts by the box as well as pies. Each pie costs \$12 and each box of donuts costs \$8. On a particular weekend, Bluebird Orchards sold 50% more boxes of donuts than pies and for a total of \$528. How many boxes of donuts were sold that weekend?

4-2 The sum of all positive values of x that are part of solutions of the system $\begin{cases} y = x^2 - 10y + 6 \\ \frac{x-5}{y+1} = \frac{y-1}{x+5} \end{cases}$ can be written as $a + b\sqrt{c}$, where a, b, and c are all integers and c has no perfect square factors larger than 1. Find $a + b + c$.

4-3 Given the system $\begin{cases} x^2 + y^3 = 42 \\ y^3 + z^4 = 104 \\ x^2 + z^4 = 98 \end{cases}$, the solution is $(a\sqrt{b}, c\sqrt[3]{d}, f\sqrt[4]{g})$ where a, b, c, d, f, and g are positive integers greater than one and all quantities are in simplest radical form. Find the value of $a + b + c + d + f + g$.

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Round 5: Right Triangles

5-1 What is the measure in degrees of an acute angle whose cotangent is three times its tangent?

5-2 A right triangle has the property that the sine of one of its acute angles is three times the cosine of the same angle. If the triangle has an area less than 375, what is the largest possible integer length of the hypotenuse of the triangle?

5-3 A right triangle whose side lengths form an arithmetic sequence has an area of 222 square units. What is the square of the length of the hypotenuse?

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Round 6: Coordinate Geometry

6-1 The point P with coordinates $(8,3)$ is rotated 90 degrees counterclockwise around the origin to make the point P'. Find the square of the distance from P to P'.

6-2 Given the points $A(2,3)$, $B(6,4)$, and $C(8,9)$, there are exactly three other distinct possible locations for point D such that the four ordered pairs form the vertices of a parallelogram. Find the sum of all possible $y-$coordinates of point D.

6-3 The graph of the equation $2\sqrt{x}-\sqrt{y}=4$ shares an ordered pair with a line with intercepts of $(0,296)$ and $(74,0)$. Find the square root of the product of the coordinates of this ordered pair.

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Match 1
Team Round

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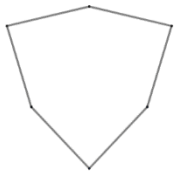
T-1 In an arithmetic sequence, the third term is 40% greater than the first term. Consequently, the 12th term is p percent greater than the first term. Find the value of p.

T-2 Find the positive integer value of k such that the equation $\frac{x}{x-3}+\frac{5}{x+8}=\frac{x^2+20x+k}{x^2+5x-24}$ has no solution for x.

T-3 Consider rectangle $FCML$, with point A on diagonal $\overline{FM}$ such tht $FA=\frac{1}{4}FM$, and point H on $\overline{CM}$ such that $\overline{AC}||\overline{LH}$. If the area of trapezoid $ACHL$ is 84 square units, find the area of rectangle $FCML$ in square units.

T-4 If the ordered pair (a,b) solves the system $\begin{cases}3x+2y=288\\ \frac{1}{12x}+\frac{1}{8y}=2\end{cases}$, find the value of ab.

T-5 An equilateral hexagon has a perimeter of 36 and has angles that alternate between measuring 90 degrees and 150 degrees (see the diagram). The furthest distance from the center of the hexagon to a point on the perimeter is $\sqrt{a}+b\sqrt{c}$, where $a, b,$ and c are integers and a and c have no perfect square factors greater than 1. Find $a+b+c$.



T-6 A circle lies in Quadrant I and is mutually tangent to the lines $y=\frac{3}{4}x$ and $y=\frac{4}{3}x$, and the x-coordinate of the point of tangency to $y=\frac{3}{4}x$ is 12. The circle has a radius of $\frac{a}{b}$, where a and b are integers with no common factors greater than 1. Find $a+b$.