FAIRFIELD COUNTY MATH LEAGUE 2021-2022

Match 6 Round 1
Arithmetic: Lines and angles

1.) {66,55,48}

2.) {54,53,59}

3.) {98,50,32}

Note: Solutions are provided for Form A only. All forms have similar solution methods.

1.) If the measure of an angle in degrees is {11,6,19} more than {one-twelth, one-fifth, one-sixth} that of its supplement, what is the measure of its complement in degrees?

2.) $\angle FCM$ and $\angle LCD$ are vertical angles for which $m\angle FCM = (x^2)^{\circ}$ and $m\angle LCD = (xy + \{44,45,50\})^{\circ}$. If x and y are both integers and $y > 0$, find the sum of the greatest possible value of x and the greatest possible value of y.

3.) Line m has equation $5x - 3y = \{42,30,24\}$. Line n has equation $5x - 3y = K$ and is exactly {7,5,4} units from line m. Find the product of all possible values of K.

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Match 6 Round 2
Algebra 1: Literal Equations

1.) $\{55,40,61\}$

2.) $\{28,84,62\}$

3.) $\{18,16,17\}$

Note: Solutions are provided for Form A only. All forms have similar solution methods.

1.) If the equation $4x - y + 3z = 2(Ax + By)$, where A and B are constants, is solved for z, the result is $z = \{2,4,6\}x + \{7,3,5\}y$. Find the value of $3A + 4B$.

2.) If $x = \{3,5,4\}y + \sqrt{4y^2 + \{7,5,11\}}$ then there exist constants $a, b,$ and c such that $x^2 + axy + by^2 = c$. Find the value of $a + 4b + 2c$.

3.) If the equation $x^2(y + 3M) = 4x(x^2 + Nx + 2) - 2(y - 5)$, where M and N are constants, is solved for y, the resulting equation can be written as $y = ax + b$ for some constants a and b. Find the value of $6M - 8N + \{2a + 4b, 4a + 2b, 3a + 3b\}$.

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Match 6 Round 3 Geometry: Solids and Volume

1.) $\{96,150,216\}$

2.) $\{43,13,28\}$

3.) $\{828,1938,258\}$

Note: Solutions are provided for Form A only. All forms have similar solution methods.

1.) Consider a right cylinder whose height is equal the diameter of its bases. If this cylinder has a volume of $\{128\pi, 250\pi, 432\pi\}$, then its surface area is $k\pi$. What is the value of k?

2.) A cone is inscribed in a hemisphere and both share a circular base. If the surface area (including the base) of the cone is $\{15,5,10\}$ square units, then the surface area of the hemisphere is $a + b\sqrt{c}$ square units, where a, b, and c are integers and c has no perfect square factors larger than 1. Find the value of $2a + 3b - c$.

3.) A cube is inscribed a hemisphere of radius $\{9,12,6\}$ units such that the center of the base of the cube rests on the center of the base of the hemisphere. The volume inside the hemisphere but outside the cube can be written in the form of $A\pi - B\sqrt{C}$ units, where A, B, and C are positive integers with C having no perfect square factors greater than 1. Find the value of $A + 2B + 3C$.

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Match 6 Round 4 Algebra 2: Radical expressions and equations

1.) {3,2,4}

2.) {6972,5852,4970}

3.) {812,870,930}

Note: Solutions are provided for Form A only. All forms have similar solution methods.

1.) What is the extraneous solution to $\sqrt{x - \{2,1,3\}} + \{4,3,5\} = x$?

2.) What is the largest value of $n \leq \{7000,6000,5000\}$ such that the expression $\sqrt{n + \sqrt{n + \sqrt{n + \cdots}}}$ evaluates to an integer?

3.) If $a_0 = 0$ and $a_n = a_{n-1} + 1 + \sqrt{4a_{n-1} + 1}$ for $n > 0$, what is the value of $\{a_{28}, a_{29}, a_{30}\}$?

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Match 6 Round 5
Precalculus: Polynomials
and Advanced Factoring

1.) $\{416, 224, 266\}$

2.) $\{660, 390, 273\}$

3.) $\{27, 32, 37\}$

Note: Solutions are provided for Form A only. All forms have similar solution methods.

1.) The function $f(x) = x^2 + ax + 24$ for some constant a has one zero of $x = \{-2, -4, -3\}$. What is $f(a)$?

2.) A polynomial equation $x^4 + ax^2 + bx = c$ where a, b, and c are integers has a solution set that includes $x = \{4 + 2i, 1 + 5i, 2 + 3i\}$ and $x = 3$. What is the value of c?

3.) The polynomial $x^4 + ax^3 + bx^2 + cx - 4$, where a, b, and c are integer coefficients, factors into $(x^2 + 4)(x^2 + (c - 3a)x + a - 2b)$. Find the value of $\{2a - b + c, 3a - b + c, 4a - b + c\}$.

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Match 6 Round 6
Miscellaneous: Counting and Probability

1.) $\{8,12,15\}$

2.) $\{69,82,95\}$

3.) $\{36000,4320,576\}$

Note: Solutions are provided for Form A only. All forms have similar solution methods.

1.) A bin contains M marbles, 5 of which are red. The probability of selecting two red marbles randomly from the bin is $\left\{\frac{5}{14},\frac{5}{33},\frac{2}{21}\right\}$. What is the value of M?

2.) A basketball player makes $k\%$ of his free-throws. He practices by throwing N free-throws. If each free-throw is independent, the probability that he will make at least n of the shots is found by computing $1 - \binom{\{10,11,12\}}{0}\{.35,.45,.55\}^0\{.65^{10},.55^{11},.45^{12}\} - \binom{\{10,11,12\}}{1}\{.35,.45,.55\}^1\{.65^9,.55^{10},.45^{11}\}$. Find the value of $2n+3N+k$.

3.) A group of $\{8,7,6\}$ students is standing in a row for a picture. The photographer does not want the three tallest students grouped together. Two of them *can* be next to each other as long as the third is not next to either one. How many ways can the students be arranged for the picture? (Assume left-to-right order matters.)

Team Round

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1.) 2022	4.) 94
2.) 1234	5.) 82
3.) 79	6.) 65

1.) A convex 20-gon has the following properties: it has A acute angles each with a measure of B degrees, C obtuse angles each with a measure of D degrees, and no right angles. Angles with measures B and D are supplementary. If $A, B, C,$ and D are positive integers, find the value of $8A + 3B + 6C + 11D$.

2.) If $z = 5x - \frac{2}{y}$ is a solution to the equation $y^2(z - Ax)(z + Ax) + Bxy = C$ for positive constants $A, B,$ and C, find the value of $40A^2 + 11B + \frac{7}{2}C$.

3.) A cylinder with a height of 10 inches and a base radius of 12 inches is sliced by a plane along the furthest distance from its top base to its bottom. (Note: the plane is diagonal such that the plane intersects each base at a single point, dividing the original cylinder into two congruent solids.) The ratio of the surface area of one of the two resulting congruent solids to that of the original cylinder can be written in simplest form as $\frac{a}{b}$ where a and b are positive integers with no common factors greater than 1. Find $a + b$. (Note: the area of an ellipse is $\pi r_1 r_2$ where r_1 and r_2 are the lengths of the semi-major and semi-minor axes.)

4.) Find the sum of all values of x such that there exists an ordered pair (x, y) where x and y are positive integers and $\sqrt{x+y} + \sqrt{x-y} = 8$.

5.) Consider the polynomial $f(x) = x^3 + 5x^2 + ax + b$, where a and b are real constant coefficients and $b < 0$. The sum of the squares of the zeros of $f(x)$ is 13 and $f(b) = 31b$. What is the value of $f(3)$?

6.) In the town of Sunnyville, days are either sunny or rainy. If it is sunny today, there is a 72% chance it will be sunny tomorrow. If it is rainy today, there is a 48% chance it will be rainy tomorrow. The overall proportion of sunny days over time in Sunnyville is p. Find $100p$.